Experience with Experimentally Modeling Structure-Actuator-Sensor Systems for Control Design

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ABSTRACT

This paper summarizes work done to experimentally characterize the combined actuator-structure-sensor system, of the Active Control Evaluation for Spacecraft (ACES) facility at the NASA Marshall Space Flight Center. The input-output frequency response functions of the actuator-structure-sensor system were measured between each of the available actuators, and all of the available analog sensor outputs. A modal model, representing the input-output characteristics of the system, was generated to use for closed loop controller design. Since the system includes various different actuators and sensors, with significant dynamics, assumptions of reciprocity, non-critically damped poles, and proper transfer function forms, are not necessarily valid. This paper looks at what is required to model such general systems, from an experimental modal analysis viewpoint.

1. INTRODUCTION

The NASA, Control Structure Interaction (CSI) program was initiated to investigate and develop methods for identification and active vibration control of large space structures. The authors have participated in the CSI program as Guest Investigators (GIs). Initial work has focussed on characterizing the NASA Marshal ACES (Active Control Evaluation for Spacecraft) facility test structure, with the goal of generating reliable models for future control system design (ref. 1, 2).

T.,e ACES structure, shown in Figure 1, is a long, flexible, vertically hanging truss, with counterbalanced arms supporting a gimbaled pointing mirror and an antenna. The pointing mirror reflects a laser beam onto an antennae, which measures x and y line of sight (LOS) error. A variety of actuators and sensors are available for use in control method implementation. These include linear momentum exchange devices (LMEDs), torque gimbals (AGS - Advanced Gimball System), a base translation table (BET), rate gyros, servo accelerometers, and LVDT position sensors. An HP/COSMEC system is used for measurement and digital control.

The actuators have significant dynamics associated with them, which must be included in the system model. These dynamics are believed to be due to the mechanical characteristics of the actuators, and also, the electronic signal conditioning and compensation.

In order to identify a model of this system which is suitable for control design, a number of issues must be considered, which are not normally relevant to modal analysis of structural systems. Assumptions of reciprocity, non-critically damped poles, and applicability of a partial fraction model, may not be valid.

2. DATA ACQUISITION

The approach taken, was to measure the characteristics of the combined actuator-structure-sensor system, as shown in Figure 2. The actuator dynamics were not measured separately, as they could not be removed from the structure, and testing in place would be difficult, if not impossible. It is believed, based on manufacturers information, and the experience of the personnel involved, that most of the ACES sensors have negligible dynamics. The LOS error detector signal, has some delay associated with it, as it is a microprocessor based sensor.

A Hewlett Packard (HP) 300 series workstation with a HP-3565 (Paragon) data acquisition system was used to acquire data and calculate frequency response functions (FRFs). Multi-input multi-output, testing was performed using up to 3 actuators and measuring 21 analog outputs. A burst random excitation technique was used to minimize leakage. Data was measured primarily over the 0 - 12.5 Hz frequency range, which includes over 50 structural modes.

3. SPECIAL CONSIDERATIONS FOR A NON-STRUCTURAL SYSTEM

3.1 Actuator Dynamics

The actuator dynamic effects resemble a pure time delay, in the 0 - 12.5 Hz frequency range. The phase of a collocated FRF measurement on a structural system varies by 180 degrees, but does not have any cumulative roll off with increasing frequency. Collocated FRFs measured on the ACEs system, show a near linear phase roll off, superimposed on the 180 degree phase variations due to the structural modes. This time delay is believed to be due to the analog processing and low pass filtering performed in the actuator electronics. A time delay accurately models the observed characteristics of the actuators, over the 0-12.5 Hz frequency band of interest.

Figure 3 shows a 0 to 12.5 Hz FRF measurement between the collocated base X torque gimbal, and rate gyro. The cumulative phase rolloff at 12.5 Hz is 245 degrees. This corresponds to a delay of 55 msec. 36 msec of delay is attributable to the COSMEC controller (the torque gimbals must be commanded through the COSMEC system), leaving approximately 20 msec delay due to the torque gimbal - rate gyro loop.

To model a time delay exactly, requires an infinite dimension model (ref. 3), however a time delay may be modeled over a limited frequency band, with a finite dimension pole zero model. One such model is called a Pade approximation (ref. 4). A Pade model consists of appropriately placed pole zero pairs. The order of this model may be chosen as large as required to obtain the desired accuracy over the frequency range of interest.

Figure 4 shows a first order Pade model of a 20 msec time delay over the 0 - 12.5 Hz frequency range. This consists of a real pole at $s=-100$ and a real zero at $s=100$, with transfer function;

$$H(s) = \frac{-(s-100)}{(s+100)}$$

A characteristic of Pade models, of any order, is the poles and zeros have large real parts. Thus, estimating a pole zero model of a these actuator dynamics, from experimental data, is synonymous to estimating the poles of an extremely highly damped system. In addition, these actuator poles occur among the lightly damped structural poles. There is little experience in the modal analysis field, in estimating models of systems with mixed lightly and heavily damped poles. Poles with large real parts are generally assumed to be computational poles (Ref. 5), and are discarded. In general, current experimental modal analysis parameter estimation methods, are not well suited to this task.

In view of this, the approach taken to analyze the ACES data, was to remove the time delays from the FRF measurements, prior to parameter estimation. This is accomplished by multiplying the measured FRFs by $e^{j\omega T}$, which is the frequency domain representation of a time shift of T seconds (ref. 7). The value of T for each actuator was estimated from the linear phase roll-off at the driving point measurement.

For control purposes, the actuator dynamics may be accounted for in different ways. If all the actuators used in the control implementation have similar dynamics, which may be approximated by a time delay over the frequency range of interest, the delay may be removed from the FRFs prior to estimating a model. The time delay is then treated as a measurement delay, and accounted for in the design of the discrete time controller. This technique was used for the ACES project.

If actuators with different dynamics are to be used simultaneously, the actuator dynamics must be included in the system model.

If the dynamics are known, and can be removed from the FRFs, as in the case of time delays estimated from driving point measurements, the structural model can be identified in a normal manner, and the actuator dynamics incorporated into the structural model to form the complete system model.

If actuator dynamics are unknown, the complete system model must be estimated directly. To do this, the identification method must have the capability to estimate highly damped poles and the associated residues. In any case, where different actuator dynamics are incorporated into the system model, a non reciprocal model results, as shown in the following section.

3.2 Incorporation of Actuator Dynamics into System Model

One of the standard assumptions of modal analysis of structures, is that the system is reciprocal. When dealing with the input-output relationships of actuator-structure-sensor systems, which have multiple actuators with different dynamics, this assumption is not valid. This is true of the open loop, uncontrolled system, as well as the closed loop system.

This may be illustrated by examining the system in Figure 5, which consists of a two DOF structure, with transfer functions H_{11} , H_{12}, H_{21}, and H_{22}, and two actuators, with transfer functions H_{A1}, and H_{A2}, representing their dynamics. I_1 and I_2 are the actuator commands, and F_1 and F_2 are the actual forces applied to the structure by the actuators. As the structure is reciprocal, $H_{12} = H_{21}$. Integrating the actuator transfer functions into the system transfer function matrix, as in Figure 6, clearly, for $H_{A1} \neq H_{A2}$, the transfer function matrix is no longer symmetric, and the system is not reciprocal.

To illustrate this in a numerical example, a proportionally damped MKC system is chosen with;

$$M = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \quad K = \begin{bmatrix} 1200 & -400 \\ -400 & 400 \end{bmatrix} \quad C = \begin{bmatrix} 1.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$$

The vector of pole values, modal participation vector matrix, modal vector matrix, modal scale factors, and residue matrices for this system, are designated by Λ, Ψ, Φ, Q, and A_r respectively. These parameters, which were calculated with Matlab (ref. 7), are given below. The modal vectors and modal participation vectors are scaled such that the first coefficient is unity.

$$\Lambda = \begin{Bmatrix} -0.39+25.12i \\ -0.39-25.12i \\ -0.11+13.00i \\ -0.11-13.00i \end{Bmatrix} \quad \Psi = \begin{bmatrix} 1.00 & 1.00 & 1.00 & 1.00 \\ -1.73 & -1.73 & 1.73 & 1.73 \end{bmatrix}$$

$$Q = \begin{Bmatrix} -.0033i \\ .0033i \\ -.0064i \\ -.0064i \end{Bmatrix} \qquad \Phi = \begin{bmatrix} 1.00 & 1.00 & 1.00 & 1.00 \\ -1.73 & -1.73 & 1.73 & 1.73 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -.0033i & .0057i \\ .0057i & -.0100i \end{bmatrix} \quad A_2 = \begin{bmatrix} .0033i & -.0057i \\ -.0057i & .0100i \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -.0064i & -.0111i \\ -.0111i & -.0192i \end{bmatrix} \quad A_4 = \begin{bmatrix} .0064i & .0111i \\ .0111i & .0192i \end{bmatrix}$$

A number of observations about this system may be made. The modal vectors and modal participation vectors are the same. For a reciprocal system, these parameters may always be scaled, such that they are the same vectors (ref. 8). The system residues are purely imaginary. For a proportionally damped structure, the residues (for acceleration or displacement data) will always be purely imaginary (ref. 8). Similarly, since the residue matrices may be constructed from the modal vectors, participation vectors, and scale factors, as in Equation 1 (ref. 8), these parameters may always be scaled such that they are purely real, or imaginary, rather than complex, numbers.

$$A_{pqr} = \phi_{pr} \psi_{qr}^{T} Q_r \tag{1}$$

Actuator dynamics are now incorporated into the system, in order to examine the effect on the modal parameters. Actuator dynamics are modeled by first order Pade models, of 20 and 40 msec time delays, for inputs 1 and 2. The actuator transfer functions are;

$$H_{A1} = \frac{-(s-100)(s+50)}{(s+100)(s+50)}$$

$$H_{A2} = \frac{-(s-50)(s+100)}{(s+100)(s+50)}$$

Note that a canceling pole zero pair is included in each actuator transfer function. This is done so that the actuator poles, are also global system poles.

The modal parameters of this system are calculated from the combined transfer function matrix, of Figure 6, and shown below;

$$\Lambda = \begin{Bmatrix} -100.00 \\ -50.00 \\ -0.39+25.12i \\ -0.39-25.12i \\ -0.11+13.00i \\ -0.10-13.00i \end{Bmatrix} \qquad Q = \begin{Bmatrix} 0.0065+0.0000i \\ 0.0015+0.0000i \\ -0.0016-0.0029i \\ -0.0016+0.0029i \\ -0.0016-0.0062i \\ -0.0016+0.0062i \end{Bmatrix}$$

$$\Psi = \begin{bmatrix} 1.000 & 0.000 & 1.000 & 1.000 & 1.000 & 1.000 \\ 0.000 & 1.000 & -1.576+0.740i & -1.576-0.740i & 1.681-0.430i & 1.681+0.430i \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 1.000 & 1.000 & 1.000 & 1.0000 & 1.0000 & 1.000 \\ 0.034 & 23.000 & -1.732 & -1.732 & 1.732 & 1.732 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0.0064 & 0.0000 \\ 0.0002 & 0.0000 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0.0000 & 0.0015 \\ 0.0000 & 0.0350 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.0016-0.0029i & 0.0047+0.0035i \\ 0.0027+0.0051i & -0.0081-0.0060i \end{bmatrix} \qquad A_4 = \begin{bmatrix} -0.0016+0.0029i & 0.0047-0.0035i \\ 0.0027-0.0051i & -0.0081+0.0060i \end{bmatrix}$$

$$A_5 = \begin{bmatrix} -0.0016-0.0062i & -0.0054-0.0097i \\ -0.0028-0.0108i & -0.0094-0.0169i \end{bmatrix} \qquad A_6 = \begin{bmatrix} -0.0016+0.0062i & -0.0054+0.0097i \\ -0.0028+0.0108i & -0.0094+0.0169i \end{bmatrix}$$

With the addition of actuator dynamics, a number of changes occur in the modal parameters. The system is no longer symmetric (reciprocal), as evidenced by the non-symmetric residue matrices, and the modal vectors and modal participation vectors which are not similar. The residues are complex valued. The modal participation vectors are complex in the sense that they cannot be scaled in such a way that they contain no complex values.

There are a number of implications for the experimental identification of systems which have different actuator dynamics incorporated in them. A modal parameter estimation method which estimates complex residues, must be used. This is the case, even if the structure has real normal modes. In addition, the method must estimate complex modal participation vectors, as they are required to define the system model.

3.3 Partial Fraction Model

Modal analysis, based on a partial fraction expansion of the system transfer function, involves implicit assumptions regarding the form of the transfer function. In order to expand the transfer function to a partial fraction form, the order of the numerator polynomial must be less than the order of the denominator polynomial (ref. 9). While this will be true for transfer functions of structural systems, it is not necessarily true for transfer functions based on the input output relationships of an actuator/structure/sensor system.

An example of this, is the transfer function between an LMED command and the collocated accelerometer response. The input to the LMED, commands the relative position between the structure and the LMED reaction mass (Figure 7). Denoting x as the relative displacement, x_1 as the displacement of the structure at the LMED location, and x_2 as the displacement of the reaction mass, the transfer function of $\frac{\ddot{x}_1}{x}$ may be derived as follows.

The force applied by the actuator to the structure is opposite to that applied to the reaction mass, which is

$$f = m\ddot{x}_2 \tag{2}$$

The displacement, x_1 is given by

$$x_1 = -H_{11} f \tag{3}$$

or

$$\ddot{x}_1 = -H_{11} f s^2 \tag{4}$$

Where H_{11} is the driving point transfer function at the LMED location on the structure. The relative displacement is

$$x = x_2 - x_1 \tag{5}$$

Substituting equations 2 and 5 into equation 4, and re-arranging, gives

$$\frac{\ddot{x}_1}{x} = \frac{-H_{11} M s^4}{1+H_{11} M s^2} \tag{6}$$

If $H_{11} = \dfrac{N(s)}{D(s)}$, where $N(s)$ and $D(s)$ are polynomials in s, equation 6 may be rewritten as

$$\frac{\ddot{x}_1}{x} = \frac{-N(s) M s^4}{D(s) + N(s) M s^{\cancel{4}\,2}} \tag{7}$$

For a structural system, the order of $D(s)$ is 2 less than the order of $N(s)$, thus the order of the numerator polynomial in equation 7 is 2 greater than the order of the denominator polynomial. This data cannot be represented with a partial fraction expansion. Additional terms are required as in equation 8 below.

$$\frac{X(s)}{U(s)} = \sum_{r=1}^{2N} \left(s^2 A_r + c_1 s A_r \lambda_r + c_2 A_r \lambda_r^2 + \frac{A_r \lambda_r^3}{s-\lambda_r} \right) \tag{8}$$

While the polynomial terms in Equation 8 appear to represent residual terms due to out of band modes, they arise for a different physical reason. Figure 8 shows an acceleration over force, driving point FRF, synthesized from an analytical MKC system. This is an improper transfer function, where the order of the numerator term is equal to the order of the denominator term. Though there are no out of band modes, the overlaid partial fraction curvefit is in error, as it requires an additional constant term to match the form of the data. This phenomenon is not normally noticed in experimental modal analysis, as there are always out of band modes, which exhibit the same effect.

The additional polynomial terms in equation 8 cannot be represented exactly in a modal model. They correspond to a direct feedthrough of the input and its first and second derivatives, to the output of the system. State space theory can incorporate a feedthrough of the input, via the D matrix, of Equation 10, but can not accommodate feedthrough of the derivatives of the input.

$$\dot{x} = Ax + Bu \tag{9}$$

$$y = Cx + Du \tag{10}$$

From the standpoint of modeling a system for control design, the polynomial portion of an improper transfer function may be approximated, with artificial out of band modes, much the same way residual terms are accommodated. The effect of incorporating these artificial modes in the system model, on the control design process, is not known.

For the ACES project, this problem was not dealt with, as it was encountered on only the 4 collocated LMED accelerometer measurements, and the effect of out of band modes on the controller design process is not known.

4. ACES MODELING

For control purposes,there were two groups of actuators on the ACES structure, which were of interest, to our colleagues doing the controller design. These were the 3 base torque gimbals (AGS), and the 4 LMED devices. The actuators within each group were similar, and their dynamics could be accurately modeled by a time delay in the frequency range of interest. Two separate models were estimated, one from FRF data with AGS excitation, and one from FRF data with LMED excitation. The time delays were removed from each data set, and normal mode models estimated, using the polyreference time domain parameter estimation algorithm (ref. 10,11).

For independent AGS and LMED controllers, the time delays which were removed from the data, are accounted for as measurement delays, in the controller design process. For coupled AGS and LMED controllers, the two models must be combined, and the different AGS and LMED time delays incorporated into the model, as in section 3.2 of this paper. Combining the two models requires some judgment, as the two sets of actuators excite some common, and some different modes. Recognizing the common modes is complicated by the nonlinearity in the structure, which causes slightly different pole values to be estimated for different excitation points.

Figure 9 shows a representative FRF, measured with LMED excitation, overlaid with the equivalent FRF synthesized from the modal model. Except for the collocated LMED accelerometer FRFs (Figure 10), synthesized data agreed well with the time shifted, measured data. This is true for both the LMED and AGS models.

5. SUMMARY AND CONCLUSIONS

With the growing interest in active control of structures, there is an great need for methods to experimentally identify combined actuator-structure-sensor models (ref. 12). Associated with this, is a realization in the controls community, that a major part of the effort in implementing a practical control system, is generating an accurate system model (ref. 13, 14).

Experimental modal analysis is a mature technology, well developed for identifying models of structural systems. The ACES CSI project provided the authors the opportunity to apply modal analysis methods to the more general actuator-structure problem. Through the course of the ACES work, a number of points became evident. In order to apply these methods to combined actuator-structure systems, they must be enhanced, to accommodate mixed lightly and heavily damped poles, nonreciprocal models, and data which cannot be modeled by a proper transfer function form.

6. ACKNOWLEDGEMENTS

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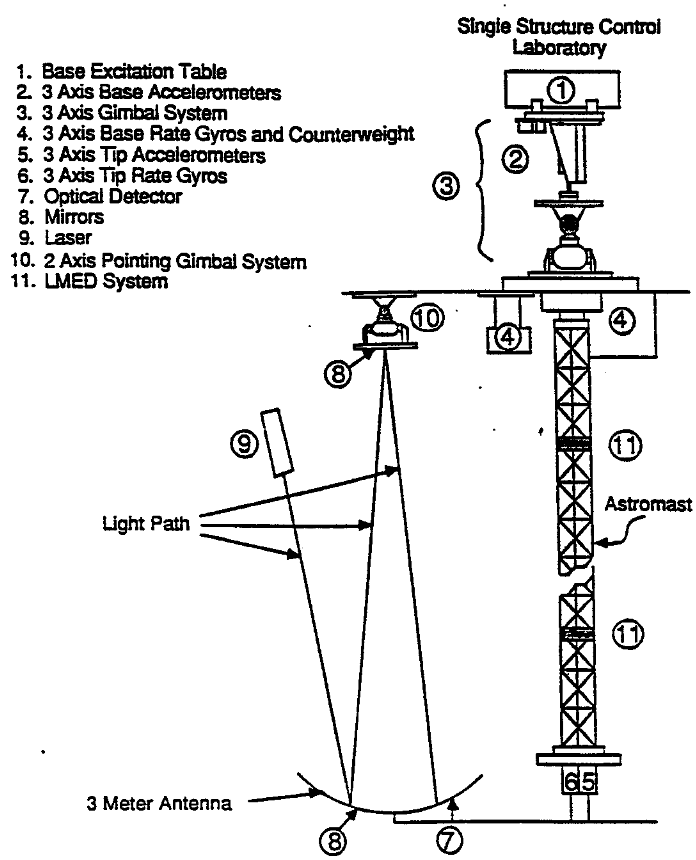

Figure 1. ACES Structure

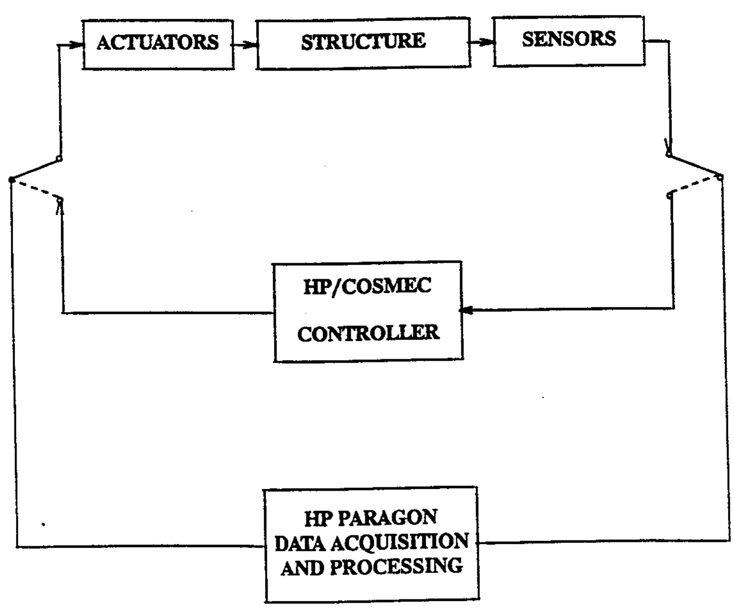

Figure 2. Data Acquisition

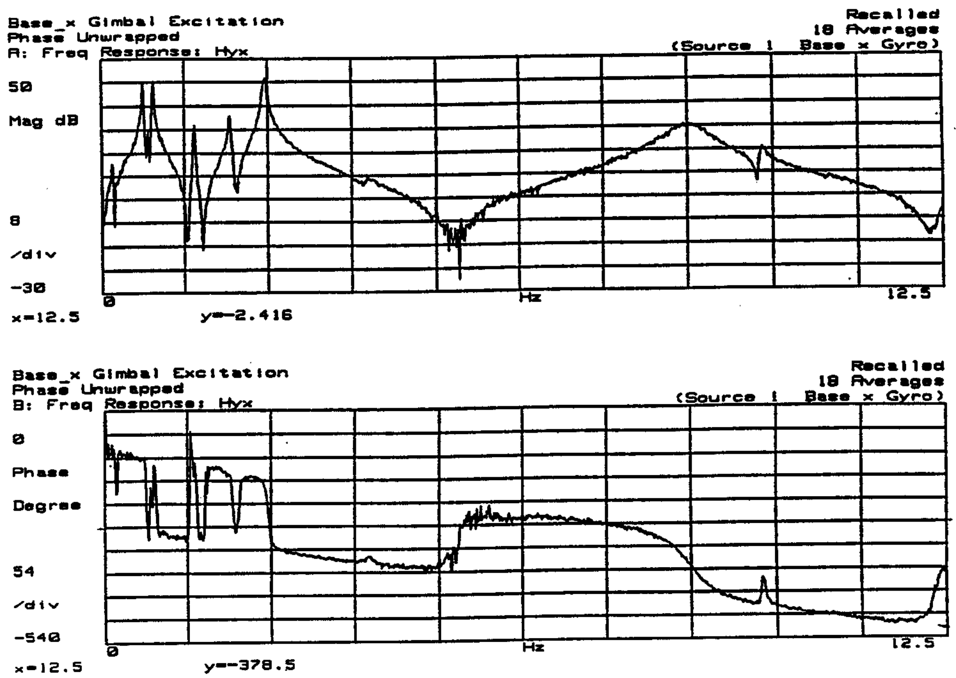

Figure 3. Time Delay due to Actuator Dynamics

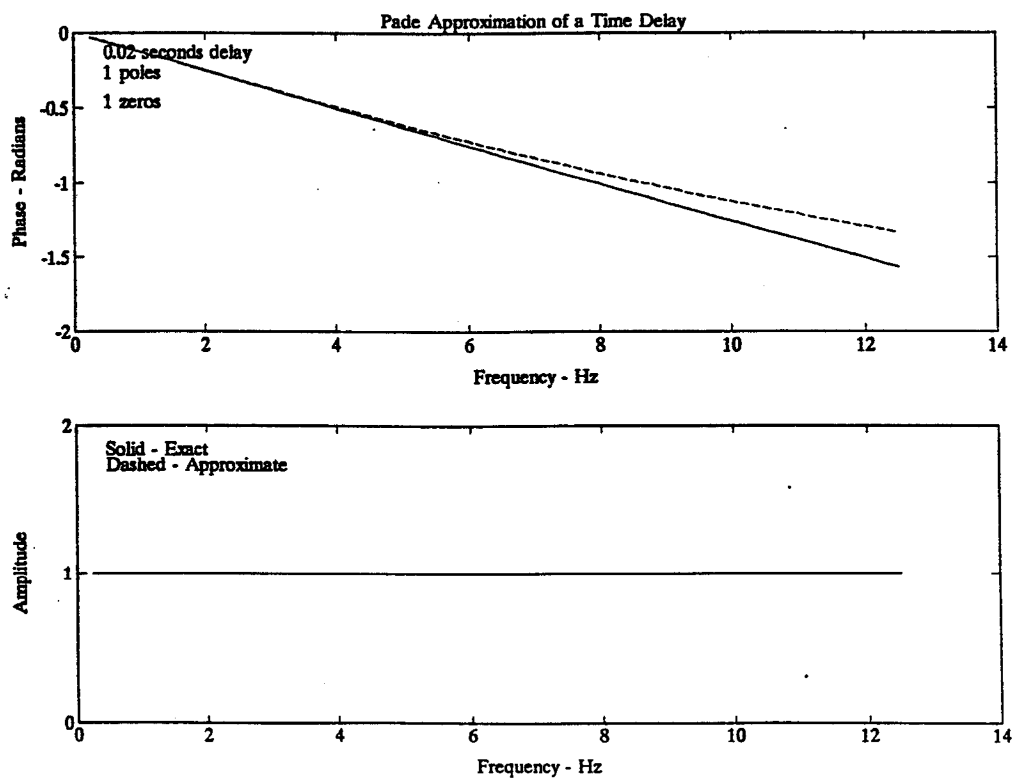

Figure 4. Pade Model of Time Delay

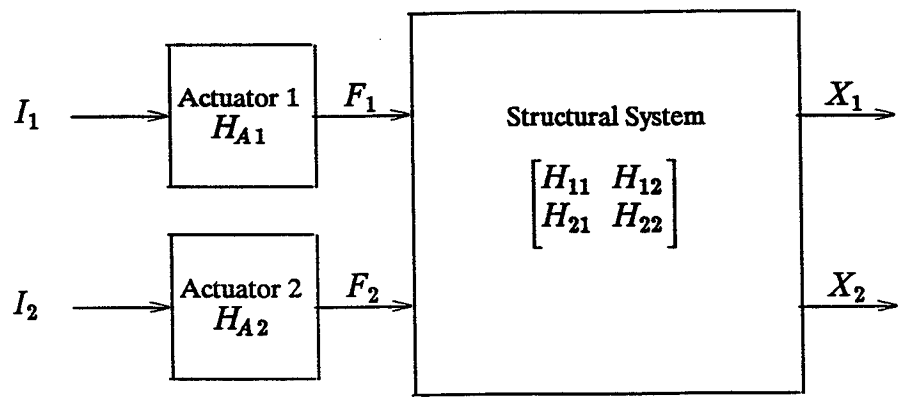

Figure 5. Actuator and Structure System

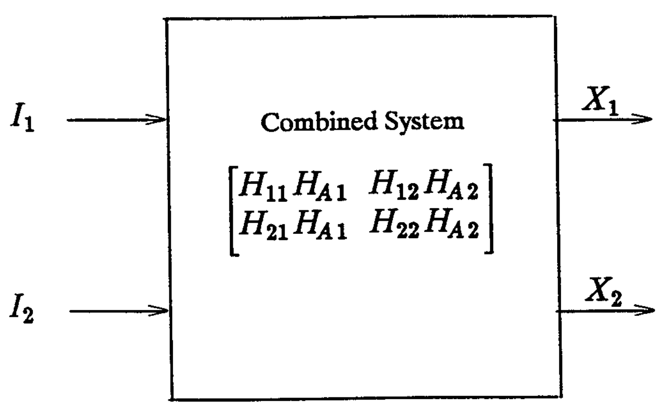

Figure 6. Combined System

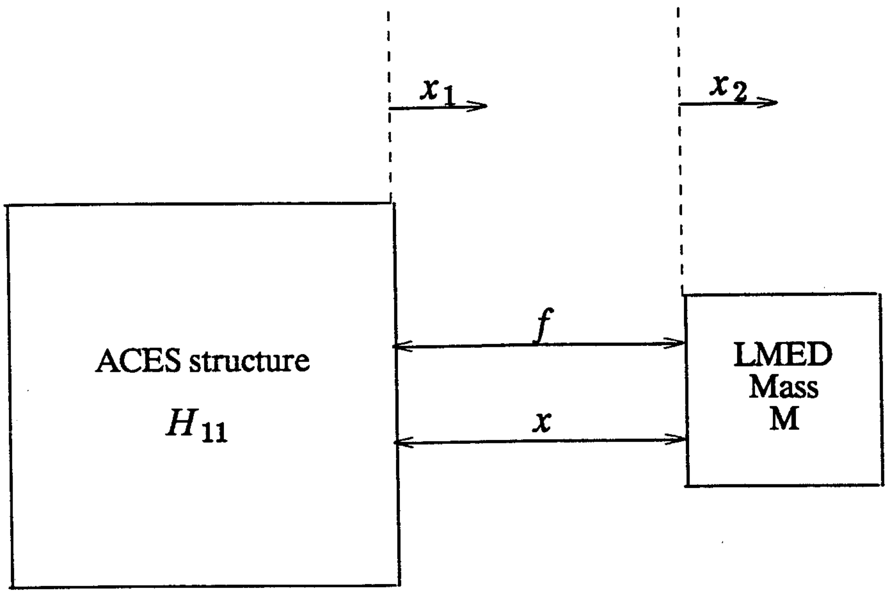

Figure 7. LMED Model

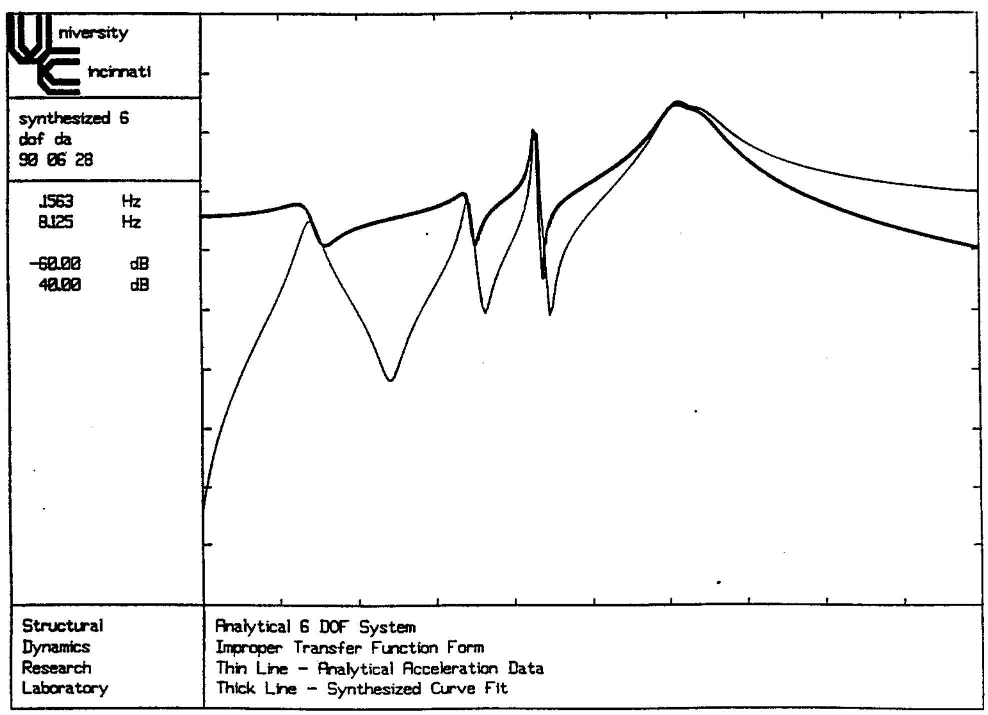

Figure 8. Improper Transfer Function