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DYNAMICS OF TETHERED SATELLITE FORMATIONS

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This paper will examine the feasibilty of multi-tether systems (MTS), i.e. satellite formations linked by a system of tethers, held taut by rotation of the entire system. This approach greatly simplifies the satellite relative navigation problem, and can eliminate the need for propulsion to maintain the formation. On the other hand, it increases the complexity involved in deploying the satellite cluster, and introduces the possibility of snags and/or slack tether. The purpose of this paper is to investigate the dynamics of tethered satellite formations, specifically for the challenging case of Earth-orbiting, Earth-facing missions. The results obtained should form the basis for techniques to quantify whether the potential benefits of tethered satellite configurations over free-flying ones outweigh their increased complexity.

NOMENCLATURE

k: number of satellites making up tether ring;
m: mass of each ring satellite;
M: mass of each anchor satellite in gravity-gradient anchored MTS;
r: radius of tether ring;
l: length of each satellite-to-satellite tether in tether ring;
h: half-height of anchor in gravity-gradient anchored MTS;
n: mean motion of MTS orbit;
ω: spin rate of MTS;
N: number of conductive wires making up geomagnetically-torqued tether ring;
i: current in each loop;
P: total power consumed;
B: geomagnetic flux density;
ρ: resistivity of tether wire;
A: cross-sectional area of each tether wire;
T: tether tensions (distinguished by subscripts).

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INTRODUCTION

There is currently considerable interest in flying many small satellites in a cluster, or swarm, having a maximum dimension on the order of hundreds of meters or a few kilometers [1]-[4]: the Air Force TechSat 21 and NASA ST5 Nanosat Constellation Pathfinder and ION-F projects are examples of such formations that are planned to be launched in the next few years. Satellite formation flight allows high resolution imaging to be carried out by means of interferometry. In addition, formations allow simultaneous measurements to be made at multiple points, as required in magnetospheric studies. Satellite clusters also lead to significant improvements in several areas over what can be accomplished using a single large "stand-alone" spacecraft: reduced cost, increased flexibility, and improved reliability.

One way in which satellite formation flight differs from traditional orbital proximity operations is that mission lengths on the order of months will usually be required. Consequently, long-term perturbation effects that can cause the formation to be pulled apart, in particular those resulting from the oblateness of the Earth, must be counteracted. In addition, the relative navigation problem (determining the exact relative positions of the various satellites) also poses significant challenges: correcting apparent position "errors" that are actually caused by orbit determination inaccuracies can lead to excessive propellant consumption. As a result, significant research effort is currently going into investigating the use of differential GPS and/or inter-satellite links for improved relative satellite navigation [5][6].

An alternative approach to the problem of flying multiple small satellites together would be to link them by a system of flexible tethers [7]-[9], held taut by rotation of the entire system. A *multi-tether system (MTS)* of this type would greatly simplify the relative navigation problem, and avoid having to use propulsion to maintain the formation. In addition, it would allow configurations that are not achievable by free-flying spacecraft, limited as they are by the nature of orbital mechanics. On the other hand, the use of tethers increases the complexity involved in deploying the satellite cluster, and introduces the possibility of snags and/or slack tether. The purpose of the work reported here is to investigate the dynamics of MTS, specifically for the challenging case of an Earth-facing formation. This research is seen as forming the basis for tools that would allow the tradeoff between free-flying and tethered satellite formations to be studied quantitatively for any given mision. This then would allow the designer to determine whether the potential benefits of tethered satellite configurations outweigh the increased complexity for a specific application.

The paper is organized as follows. This work is first put in context with the several other papers to date [10]-[14] on various configurations of spinning tethered satellite formations. This is followed by a discussion of the dynamics of the particular problem that will be addressed, namely that of an Earth-facing formation in low Earth orbit. The difficulties that arise in this case (relative satellite motion caused by orbital effects, and the need to constantly apply torque to the formation to keep it Earth-facing) are then described, and several candidate tethered formation configurations introduced. It is shown that propulsive torquing to maintain Earth pointing is prohibitive, leading to two possible approaches: using a conductive ring tether to produce a useful torque be means of geomagnetic interaction, or using a configuration that is "anchored" by the gravity-gradient effect. Both of these approaches are analyzed, and the conclusions so obtained validated by means of simulation results.

PROBLEM BACKGROUND

A great deal of literature exists (see, for instance, Refs. [7], [8], and and the papers in [8]) for what could be termed "classical" tether missions, i.e. ones for which two spacecraft are connected by a single tether. Such a configuration tends to take up a radial equilibrium orientation, as a result of gravity-gradient forces, and has uses for power generation, spacecraft boost or deorbit, etc.. Much less study has been addressed to formations of tethered satellites, where the entire formation is generally spinning, in order to provide centrifugal stiffening. The spin axis of such a configuration will have a tendency to remain inertially fixed, making such an MTS well suited to astronomical interferometry missions. If the formation is placed in heliocentric orbit, the reduced environmental disturbance torques improve its inertial pointing performance: for this reason, the proposed tethered Submillimeter Probe of the Evolution of Cosmic Structure (SPECS) mission described by Quinn and Folta [10] is envisaged as flying at the Sun-Earth L_2 Lagrange point. Quadrelli [11] describes similar heliocentric inertially-pointed astronomical MTS configurations, as well as a librating classical tether in low Earth orbit (LEO) for synthetic aperture radar Earth observation.

The behavior of spinning MTS in low Earth orbit is made quite complicated by the presence of orbital effects that tend to shift the relative positions of the various satellites. The papers of DeCou [12][13] examined this problem for a geocentric astronomical observation mission: this was again intended to remain essentially inertially fixed, which was achieved by using a relatively high spin rate. Tragesser [14] examined a low-Earth orbit MTS intended for Earth observation: the goal was to find a stable equilibrium configuration, with a lower spin rate, with a spin axis possessing a component along the nadir direction. (The analysis given in Ref. [14] was based on the Likins-Pringle relative equilibria for rigid-body spacecraft, and it was found that these equilibria did not provide a complete description of the behavior of the inherently flexible-body MTS. Similar conclusions were reached concerning the applicability of rigid-body, and indeed free-flying satellite formation, techniques to MTS during the course of the present work.)

In the analysis that will be presented in this paper, the MTS spin axis will be required to lie along the nadir direction, rather than simply have a component along it. A fundamental constraint of the Earth-facing MTS problem is that no slack tethers are permissible: if any tether does go slack, it can no longer exert forces on its tip satellites to prevent the relative motion that is driven by orbital effects. In this way, the present problem is virtually the converse of that which arises with the proposed Air Force Research Laboratory PowerSail project [15][16]: this mission involves formation flight of two spacecraft, one the instrument-carrying bus and the other its power-generating solar array. The spacecraft are connected by a power conductor, which was originally proposed to be a tether. The goal was then to perform formation-keeping burns on one or both of the spacecraft so as to keep the tether permanently slack, so communicating no vibrational dynamics to the bus.

In the LEO application, the basic role of the tethers in an MTS can be thought of as providing long-term formation stability, so avoiding the periodic use of propellant that is required in satellite formation flight. The bulk of this propellant consumption is required to null the differential nodal drift that is caused by the oblateness of the Earth, and that tends to pull satellite formations apart along the out-of-plane direction over periods of weeks or months. The tethers in an MTS serve to keep the satellites together naturally, so avoiding the need for this long-term propulsive formation maintenance.

EARTH-FACING MTS IN LOW ORBITS

As already noted, the class of Earth observation missions in low orbits appear to present the greatest dynamics challenges. The two reasons for this are as follows: firstly, the significant relative accelerations between the spacecraft that are caused by orbital mechanics, and secondly, the need to constantly torque the formation to keep it facing the Earth. These two effects will now be discussed in more detail.

The motion of a satellite relative to a nearby circular orbit (taken here as the orbit of the mass center of the MTS) with mean motion n is described by the Clohessy-Wiltshire (CW) equations [17]

$$\begin{aligned} \ddot{x} &= 2n\dot{z} + a_x, \\ \ddot{y} &= -n^2 y + a_y, \\ \ddot{z} &= 3n^2 z - 2n\dot{x} + a_z, \end{aligned} \tag{1}$$

where the local vertical/local horizontal (LVLH) coordinate system has the x axis directed along the velocity vector of the reference orbit, y along the orbit anti-normal, and z along the downward radius. The components a_x, a_y and a_z encompass all non-gravitational accelerations: in the present application, these will be dominated by the tether tensions. Note that the third, radial, equation has an orbital term proportional to the relative radial position z: this is the cause of the tension in a "classical" tether, which tends to align it with the local radius.

A key factor in the design of an MTS is the choice of the spin rate ω that is used to provide centrifugal stiffening to the tethers. This rate is most naturally expressed as a multiple of the orbital angular rate n, or approximately 0.001 rad/s for LEO. The reason for expressing the spin rate in terms of n can be seen by examining the accelerations that act on the individual satellites: the CW orbital acceleration terms that depend on relative distance are proportional to n^2, while the accelerations that result from the spin of the MTS are proportional to ω^2. Consequently, the ratio ω/n serves to quantify the relative importance of spin-induced to orbital effects on the dynamics of an MTS in LEO. Values of $\omega/n >> 1$ (the *fast spin* case) will lead to dynamics that are dominated by spin, with very little orbital effects. The spin axis of such an MTS will therefore tend to remain inertially fixed, making it well suited to Earth-orbiting astronomical missions [12][13], but not for Earth observation. (Heliocentric MTS [10][11] are also necessarily fast spin configurations, as n is extremely low for such orbits.) Selecting $\omega/n << 1$ (the *slow spin* case) will lead to dynamics that are dominated by orbital effects, with very little spin stiffening of the tethers: such a configuration is likely to experience difficulties with slack tethers. Consequently, choosing ω/n to be comparable to unity (*medium spin*) appears to be a good approach for the Earth observation mission class.

Several basic MTS configurations will be examined here for Earth-facing missions in low orbit. Fig. 1 shows a *tether ring*, where all satellites nominally lie in the plane perpendicular to the local orbital radius, with tethers running circumferentially between adjacent *ring* satellites. Fig. 2 then shows a *tether spoke* configuration, which is a similarly planar arrangement, but with tethers now instead leading from each satellite to a central one. Finally, Fig. 3 shows the more complicated, three-dimensional *gravity-gradient anchored* configuration. This can be thought of as a tether ring with the addition of a central *spine* tether (along the orbital radius), *anchor* satellites at each end of the spine, and diagonal tethers running between the ring satellites and each anchor. This addition seeks to exploit the gravity-gradient effect to "anchor" the MTS in an Earth-facing attitude; the details will be returned to later in the paper.

Consider now an Earth-facing MTS, spinning counter-clockwise as viewed from above (i.e. with angular momentum vector along the radially outward LVLH $-z$ axis). In order to keep this configuration facing the nadir rather than inertially fixed, a torque must continually be applied about the velocity direction. This torque could be generated propulsively, but would lead to very short mission lifetimes. To quantify this, suppose that the MTS consists of k ring satellites, held at radius r by tethers that are assumed for simplicity to be massless. If this system spins at rate ω, its angular momentum vector has magnitude $H = I\omega$, where $I = kmr^2$ is the axial moment of inertia. The constant torque that is required in order to precess this angular momentum vector at the orbital rate n is therefore

$$\tau = nH = kmr^2\omega n. \tag{2}$$

If this torque were applied by k thrusters of force f firing in the MTS tangential direction, one on each of the ring satellites, then the required continuous thrust would be given as $f = mr\omega n$. The relative mass flow rate of the propulsion system on each satellite is therefore

$$\dot{m}/m \approx r\omega n/i_s g, \tag{3}$$

where i_s is the thruster specific impulse (in s). Note that $\dot{m}/m$ increases with orbital rate and both the radius and spin rate of the MTS, but is independent of the number and mass of the ring satellites.

As an example, consider a fairly small ring of radius 500 m made up of 4 satellites of mass 25 kg each, in LEO (orbital rate 0.001 rad/s) and spinning at a rate of 0.002 rad/s. If the ring is torqued propulsively using pulsed plasma thrusters with an assumed specific impulse of 1500 s, the daily propellant consumption of each satellite would be approximately 0.6% of its initial mass. Assuming (generously) that 20% of the mass of each satellite consists of propellant, this would only allow a mission lifetime of around 1 month.

Consequently, if extended mission durations are to be feasible for multi-tether systems in LEO, it is necessary to exploit the environmental effects that are present. Four candidates exist: gravity-gradient, atmospheric drag, solar radiation pressure, and interaction with the geomagnetic field. Making use of gravity-gradient effects requires the addition of a radial anchor to the ring configuration (as shown in Fig. 3), and will be considered in detail below. Of the other three candidates, it is clear that drag cannot achieve the desired goal: this force always acts in the anti-velocity direction, and so cannot generate a torque about the velocity vector as required.

Solar radiation pressure could, in principle, generate the desired torque; however, there are severe problems with this approach that render it impractical. Suppose reflective surfaces extend radially out from each satellite, making an angle to the ring plane; the geometry is analogous to the blades of a propeller, with the satellites forming the blade roots. Then, if the Sun is in the ring plane and directly ahead or behind the ring, the upward solar force on one side satellite and the downward force on the opposite one would generate a torque along the anti-velocity vector as desired. However, two difficulties of this approach are: firstly, undesirable torques are generated when the Sun is in other orientations; secondly, no torque at all is generated when the ring is in eclipse, so preventing the ring from remaining Earth-facing during these periods. In addition, the required blades areas are prohibitively large: for the same ring parameters as before, each blade must have an area on the order of 30,000 m^2. This required area, which is proportional to total system inertia, and hence to satellite mass, is clearly impractical. This leaves electrodynamic torquing as the only remaining candidate for simple tether ring configurations: this is the subject of the next section.

ELECTROMAGNETIC TORQUING

Consider a tether ring MTS of the type shown in Fig. 1, and suppose that a current is passed through the circumferential tether that connects the satellites. (The tether spoke configuration of Fig. 2 would not allow this approach to be taken. Consequently, even though the tether spoke configuration would perhaps be somewhat simpler to deploy than the tether ring, it will not be considered further here.) This current generates an electromagnetic moment that is directed along the normal to the ring plane; if the MTS is in a near-equatorial orbit, so that the geomagnetic flux density vector is approximately along the orbit normal, the resultant torque vector will be aligned with the velocity vector, as required. This class of orbit is therefore optimal from the point of view of electromagnetic torquing.

We now wish to derive parametric equations for the required ring tether current and total power consumption, based on certain simplifying assumptions. These assumptions are that the ring can (initially, at least) be modeled as a rigid body; the spin rate is high enough that orbital effects can be neglected; the mass of the tethers is negligible compared with that of the satellites; and the ring is in a circular orbit in the geomagnetic Equator. It will be shown that, even though these assumptions are fairly severe, the expressions so obtained agree quite well with the results of a more realistic simulation model.

A tether ring made up of k satellites at a radial distance r has total area $A_{ring} = \frac{1}{2}kr^2\sin(2\pi/k)$. If the tether consists of N parallel turns of conductive wire, each with a current i passing through it, the resulting electromagnetic torque will then be $\tau = NiA_{ring}B = \frac{1}{2}Nkr^2 iB\sin(2\pi/k)$, where B is the geomagnetic flux density. If this is to keep the ring Earth-facing, it must equal the torque expression given by Eq. (2). Rearranging then gives the required current as

$$i = \frac{2mn^2(\omega/n)}{NB\sin(2\pi/k)}. \tag{4}$$

Note that i is proportional to the satellite mass and the ring spin rate, but is independent of ring radius (the required torque is proportional to the ring axial moment of inertia, and thus to r^2; however, so is the area of the ring, and thus its electromagnetic moment, leaving i independent of r). Also, increasing the number of wires making up the tether, N, reduces the required current, as expected. Finally, the effect of changing the orbital radius R_o is rather interesting. Since $n^2 \propto R_o^{-3}$, it might be thought that going to a higher orbit would reduce the required current. However (assuming a simple dipole model for the geomagnetic field), $B \propto R_o^{-3}$ also, rendering i independent of orbital altitude.

The total electrical power consumed is given as $P = NRi^2$, where $R = \rho kl/A$ is the resistance of each wire (of cross-sectional area A), and $l = 2\sin(\pi/k)r$ is the distance between adjacent satellites. This reduces to

$$P = \frac{1}{N}\frac{2\rho r[mn^2(\omega/n)]^2}{B^2A}\frac{k}{\sin(\pi/k)\cos^2(\pi/k)}. \tag{5}$$

As noted for i above, P is independent of orbital altitude; however, it does depend on ring radius, since tether resistance does. In addition, P decreases as N increases, so the total power consumption can indeed be decreased by using multiple wire turns. However, it should be noted that the total tether mass is proportional to NA, so P is inversely proportional to this mass: reducing power is accomplished at the expense of increasing tether mass. Note finally that the last expression on the right-hand side of Eq. (5) is roughly constant for 3, 4 or 5 satellites, but increases significantly for k above this range; it tends to k^2/π for high k. Consequently, large numbers of ring satellites are inefficient from the point of view of electromagnetic torquing.

As was already noted, this analysis was based on treating the ring as a rigid body. However, in reality, the electromagnetic forces on each individual tether segment will cause them to deflect as the ring rotates: the tether on the right-hand side (as viewed from above) will deflect up, that on the left will deflect down, and those forward and aft will essentially not deflect at all. Modeling each tether as a string of uniform mass per unit length μ, its lateral deflection z varies with distance d from one end as given by the string equation

$$\mu\frac{\partial^2 z}{\partial t^2} = T\frac{\partial^2 z}{\partial d^2} + f, \tag{6}$$

where f is the applied force per unit length, and T is the tether tension. Assuming the natural frequencies of the tether are much higher than the MTS spin frequency, the solution to this equation will approximately reach its steady-state expression, satisfying $\partial^2 z/\partial d^2 = -f/T$. We will therefore have $z(d) = (f/2T)d(l-d)$, which has a maximum value (at the midpoint of the tether) of $z_{\max} = (f/8T)l^2$.

But the electromagnetic transverse force per unit length on each side tether is $f = \pm Bi$, and the tether tension required to keep the satellites in their equilibrium positions as a result of the MTS spin is $T = mrn^2(\omega/n)^2/[2N\sin(\pi/k)]$. Substituting these into the expression for $z_{\max}$ and simplifying then gives

$$z_{\max} = \pm\frac{\tan(\pi/k)}{2(\omega/n)}l. \tag{7}$$

This can be seen to be inversely proportional to the spin rate ω, despite the fact that the current, and hence the transverse load, is proportional to ω. The reason for this is that the tension in the tether, which tends to prevent it from deflecting laterally, is actually proportional to ω^2.

We now present some numerical results. Consider first the same tether ring considered previously (four 25 kg satellites; ring radius 500 m; spin rate $2n$), with 5 turns of 1 mm diameter copper wire making up the tether. The resulting current is found to be 1.03 A, and the power consumed 330 W. These numbers agree fairly closely (to within around 10%) with the results that were obtained from a Simulink MTS simulation that included both orbital and tether structural effects, despite that fact that these effects were neglected in deriving Eqs. (4) and (5). (Note that all simulation results reported here used a tether mass per unit length of 3.3×10^{-3} kg/m and a stiffness, EA, of 47,311 N: these typical values were taken from [18].) Furthermore, Eq. (7) predicts a maximum midpoint deflection of 350 m, which again was in good agreement with the simulation results.

One interesting point that was revealed by simulation is that a long-term instability exists for a ring with $k = 4$, whereby the ring tends to eventually collapse to a line. The reason for this instability, which is not observed for other numbers of ring satellites, is that the moment of inertia of a ring with four satellites is independent of the vertex angle at any of the satellites. Consequently, the rotational energy $E = H^2/2I$ for a given angular momentum H is independent of this angle. By contrast, for instance, a ring with $k = 6$ has maximum moment of inertia when the ring forms a regular hexagon: this is therefore the minimum energy state that the system will tend to remain in.

As a result of this instability, simulations to study the long-term behavior of an electromagnetically torqued ring tether focused on a LEO ring with 3 satellites of mass 500 kg: the ring radius was 1 km and the spin rate $10n$. Fig. 4 shows the resulting LVLH z displacement of one of the satellites from the nominal horizontal ring plane for two cases: with no torquing (large curve), and with electromagnetic torquing (small curve). The large excursions that occur without torquing are a result of the fact that this fast spinning MTS tends to remain inertially fixed, giving LVLH z excursions of $\pm r$. By contrast, applying the correct current (3.2 A in this case) keeps the satellites within around ±50 m of the desired ring plane. (This performance could be improved further by application of active current control.) Finally, Fig. 5 shows a side view of the tether ring, illustrating the upward deflection of one side tether and the downward deflection on the opposite side. The observed deflection agrees well with the value of 150 m that is predicted by Eq. (7).

It can be seen that the current and power requirements for electromagnetic torquing of a satellite ring are generally quite high, making this approach perhaps of marginal utility in practice except for relatively small, slow-spin MTS. It therefore appears that the gravity-gradient anchored approach is the most promising for LEO multi-tether systems. This will be studied in the next section.

STABILITY CONDITIONS FOR GRAVITY-GRADIENT ANCHORED MTS

One way in which a multi-tether system (MTS) in low Earth orbit (LEO) can be made to remain Earth-facing is by exploiting the gravity-gradient effect. A geometry that allows this was depicted in Fig. 3: this shows four ring satellites, of mass m, in an Earth-facing ring, together with two anchor satellites, of mass M ($> m$), at the ends of a radial anchor of total height $2h$ ($> 2r$). Each ring satellite is connected by tethers to its neighboring ring satellites, and by diagonal tethers to the two anchors; in addition, a spine tether runs between the two anchor satellites. (The importance of this tether will be discussed shortly.) As before, the MTS is assumed to be in a circular orbit of angular rate n, and rotates (counter-clockwise, as viewed from above) about the spine at rate ω.

The design problem for this type of gravity-gradient anchored MTS involves finding combinations of spin rate, tether lengths and satellite masses for which the configuration remains stable. There is, in addition, the control problem of adjusting tether tensions (by reeling the tethers in and out by small amounts) so as to apply the forces that are needed in order resist the orbital effects that tend to pull the satellites out of position. These forces vary as the MTS rotates; a key parameter is the angle θ through which a particular ring satellite has rotated from the velocity vector. Making certain simplifying assumptions, expressions can be derived for these tensions that also allow the desired stability conditions to be obtained. These results will now be outlined.

The assumptions made in this analysis are that all tethers are massless and remain straight throughout. These tethers must counter the relative orbital accelerations, modeled by the CW equations, that tend to pull the ring satellites out of their desired simple rotational motion. In addition, the diagonal and spine tethers must cancel the radial gravity-gradient forces, of magnitude $F_{gg} = 3Mn^2h$, that act on the anchor satellites. Under these assumptions, a significant amount of algebra eventually yields the expressions

$$T_{ring}(\theta) = \frac{mrn^2}{2\sqrt{2}}\left\{2(\omega/n)^2 - \left[1 + \sin 2\theta + 3(M/m)(1-\sigma)\right]\right\} \tag{8}$$

for the tensions in the ring tethers and

$$T_{diag}(\theta) = \frac{mrn^2}{4\lambda}\left\{4(r/h)(\omega/n)\sin\theta + \cos 2\theta + 3(M/m)(1-\sigma)\right\} \tag{9}$$

for those in the upper diagonals, where $\lambda = r\big/\sqrt{r^2+h^2}$ (i.e. $\lambda = (r/h)\big/\sqrt{1+(r/h)^2}$), and σ is the fraction of the gravity-gradient force F_{gg} that is taken up by the spine tether. The tensions in the lower diagonals are given by Eq. (9) also, with a 180 deg phase-shift.

These equations can be used in the following way to solve the MTS design problem. An MTS in which any of the tethers periodically become slack will experience stability problems, since a slack tether is no longer capable of applying the required forces to its end satellites. If this problem is to be avoided, the tensions in all tethers must remain positive at all times. It can be shown that applying this requirement to Eq. (8) implies the following lower limit on the normalized spine tension σ:

$$\sigma \geq \sigma_{\min} \equiv 1 - \tfrac{2}{3}\left[(\omega/n)^2 - 1\right]\big/(M/m). \tag{10}$$

A physical interpretation of this condition is as follows: if the spine tether carries too low a tension, too much is carried by the diagonals. These then apply more radially inward force on the ring satellites than would be required to balance the desired centrifugal force, causing the ring satellites to collapse inwards and the ring tethers to go slack.

The corresponding condition for no slack diagonal tethers, from Eq. (9), is an upper bound on σ:

$$\sigma \leq \sigma_{\max} \equiv 1 - \tfrac{1}{3}[1 + 4(r/h)(\omega/n)]/(M/m). \tag{11}$$

The physical interpretation of this is that, if the spine tether tension is too high, the total inward force exerted on each anchor by the spine and diagonal tethers will exceed its outward gravity-gradient force. The anchors will therefore be pulled towards the ring plane, leading to slack diagonal and spine tethers.

Fig. 6 shows the range of normalized spin rates ω/n and normalized spine tensions that satisfy Eqs. (10) and (11), i.e. that will lead to stable MTS configurations. Note that for the numerical values used for this example (ring satellite mass 25 kg, anchor satellite 300 kg, ring radius 1 km, anchor half-height 10 km), no spin rate lower than 1.33 n can lead to a stable configuration; at this critical spin rate, only one specific value for σ gives stability. By contrast, if the spin rate is at least 4.36 n, stability can be achieved without a spine tether: the spin rate is now sufficient to hold the ring tethers in tension, despite the large inward forces on the ring satellites that are produced by the diagonal tethers.

Expressions can be obtained from Eqs. (10) and (11) for these two limiting spin rate values. Setting $\sigma_{\min} = \sigma_{\max}$ gives the lowest spin rate for which any stable solution exists as

$$\omega/n = (r/h) + \sqrt{1.5 + (r/h)^2}\,. \tag{12}$$

Note that this depends only on the geometry of the MTS, not on the masses of the satellites. Fig. 7 plots the resulting values for realistic values of r/h.

Setting $\sigma_{\min} = 0$ similarly gives the lowest spin rate for which a stable no-spine solution exists:

$$\omega/n = \sqrt{1 + 1.5(M/m)}\,. \tag{13}$$

This expression, by contrast to Eq. (12), depends only on the satellite mass ratio. Fig. 8 shows the limiting spin rates for reasonable values of this ratio.

Finally, Fig. 9 gives the ring (dashed) and diagonal (solid) tether tensions as a function of rotation angle for the same MTS masses and dimensions as before and the stable configuration (cf. Fig. 6) produced by choosing $\omega/n = 2$ and $\sigma = 0.9$. It can be seen that these tensions do indeed remain positive throughout. In addition, the ranges between their maximum and minimum values are not large, so the adjustments in tether length that would be required in order to control these tensions precisely are quite small. In simulations to date, the undeflected deployed lengths of all tethers have been held constant, for simplicity, and good results have been obtained: stable MTS design parameters lead to stable configurations that exhibit only small periodic oscillations of the individual satellites from their ideal positions. Fig. 10 illustrates this by plotting the LVLH z deflection of one of the ring satellites for the same MTS parameters as before: this deflection can be seen to remain small in relative terms throughout, corresponding to a tipping of the ring plane of at most 2 deg.

CONCLUSIONS

This paper has examined the feasibilty of multi-tether systems (MTS), i.e. satellite formations linked by a system of tethers, held taut by rotation of the entire system. This approach greatly simplifies the satellite relative navigation problem, and avoids having to use propulsion to maintain the formation; on the other hand, it increases the complexity involved in deploying the satellite cluster, and introduces the possibility of snags and/or slack tether. The paper focused on the particularly challenging case of an MTS in low Earth orbit that must be kept Earth-facing throughout, and investigated the dynamics of two configurations that allow this to be achieved non-propulsively: a planar ring of conductive tether that exploits geomagnetic interaction, and a three-dimensional geometry that makes use of gravity-gradient anchoring. In both cases, the effects of the various MTS design variables (satellite masses, tether lengths and, crucially, spin rate) were investigated, leading to insight into the MTS design problem. Finally, the analytical expressions that were obtained were confirmed by means of simulation results.

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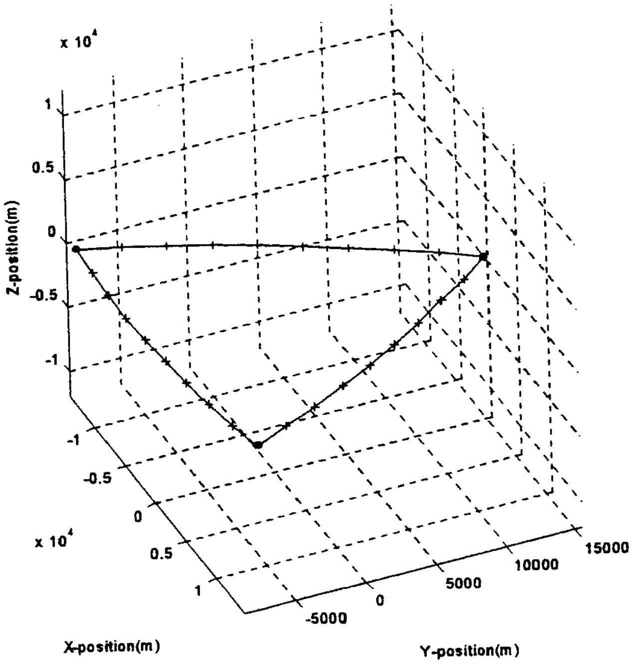

Figure 1 Tether Ring Configuration

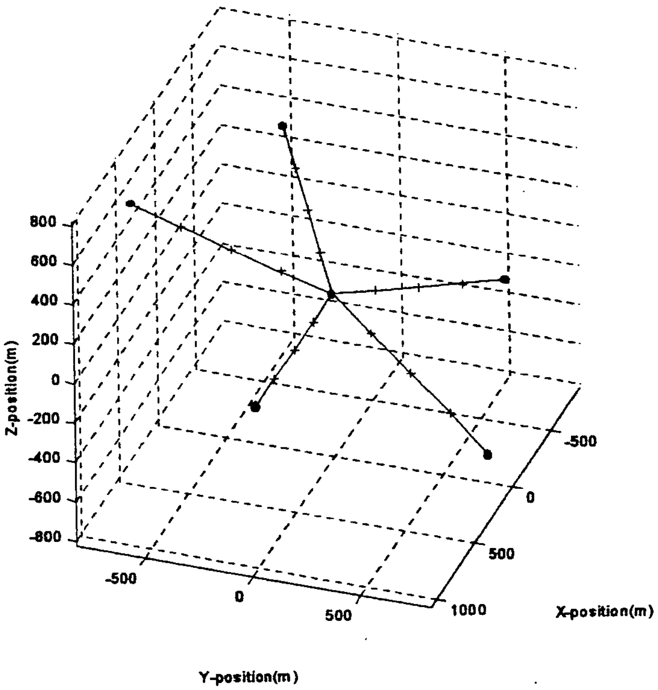

Figure 2 Tether Spoke Configuration

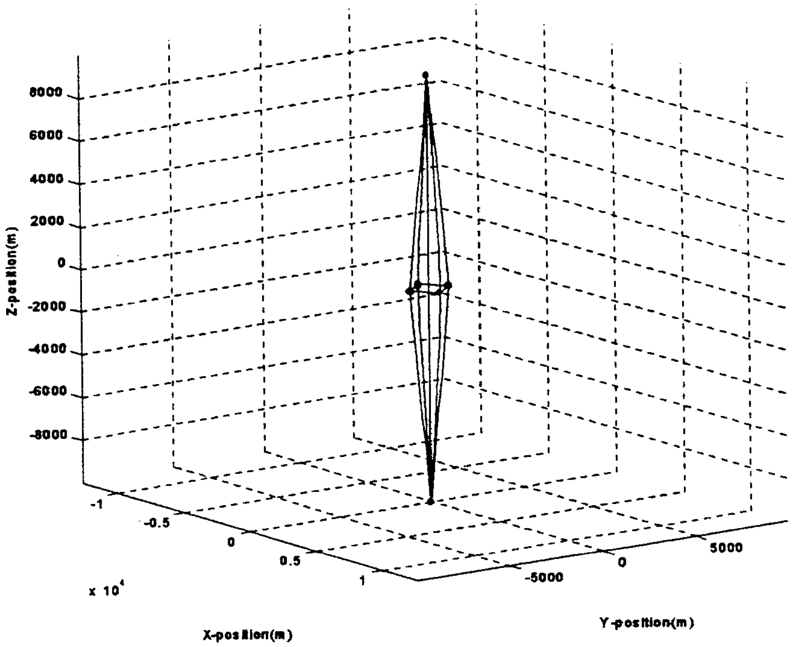

Figure 3 Gravity-Gradient Anchored Configuration

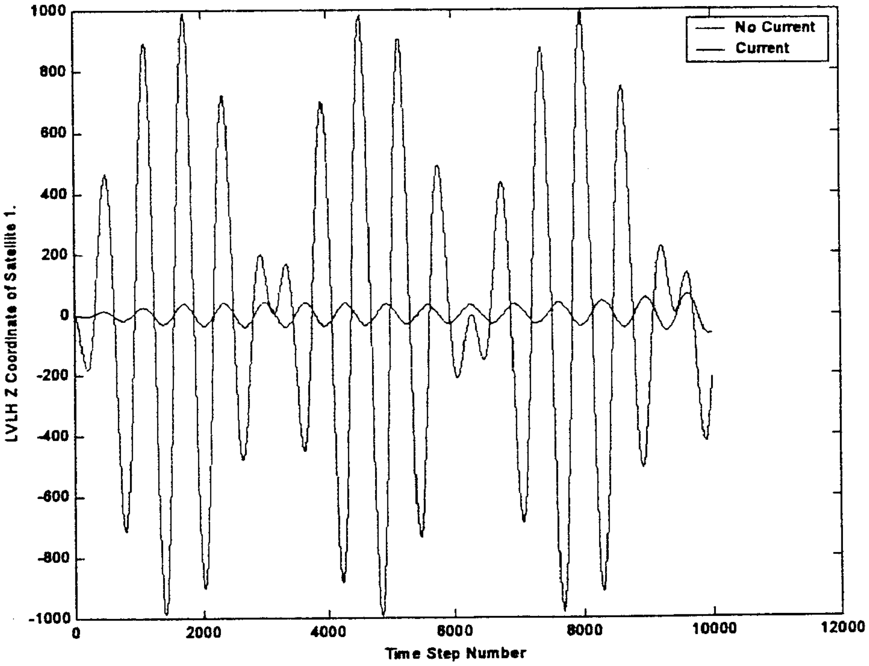

Figure 4 Electromagnetic Torquing: Satellite z Positions

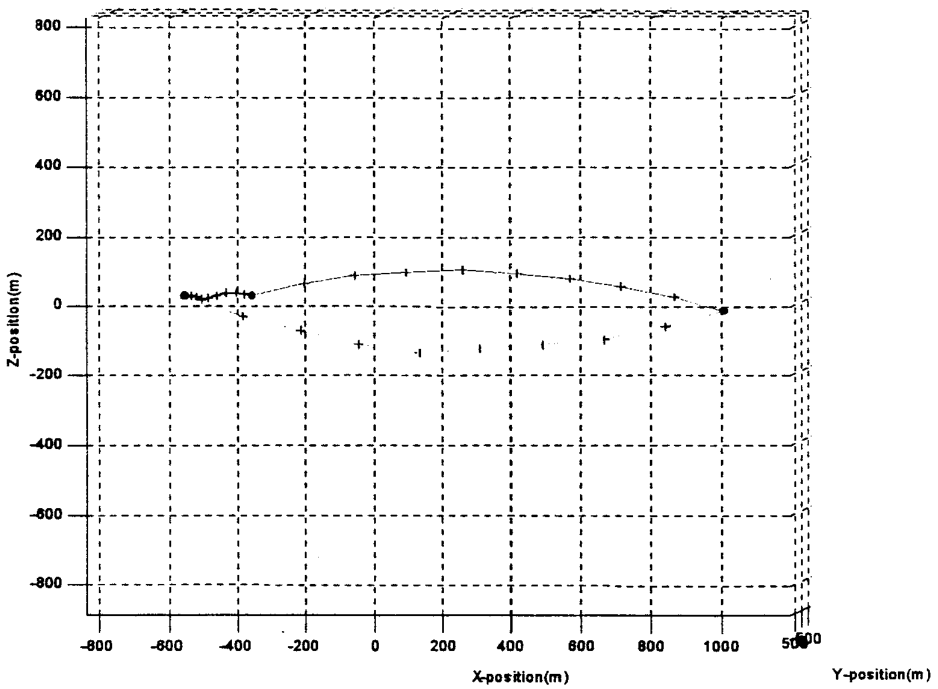

Figure 5 Electromagnetic Torquing: Tether Deflection

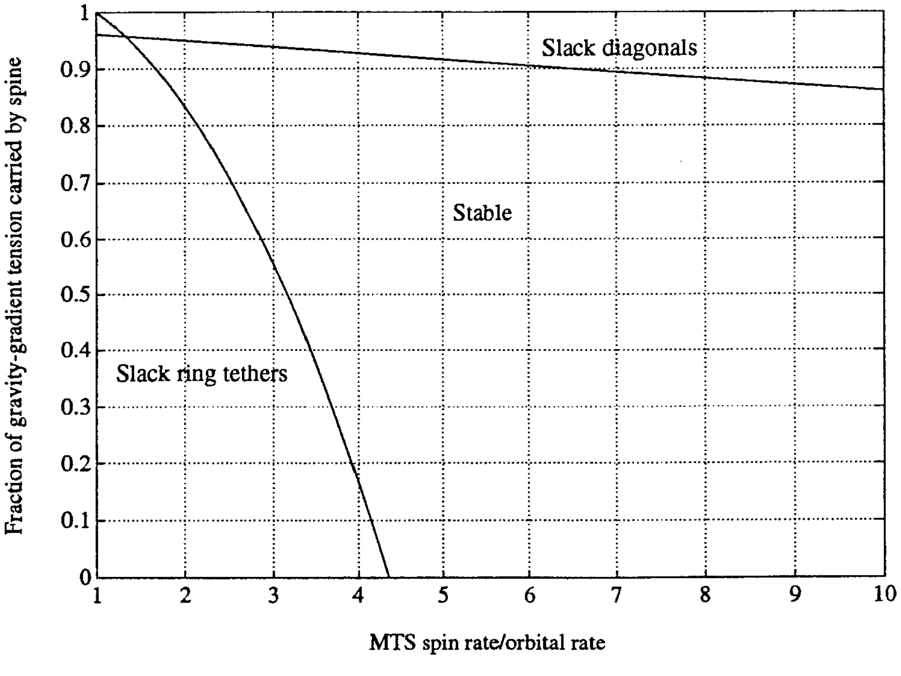

Figure 6 Gravity-Gradient Anchored Configuration Stability Region

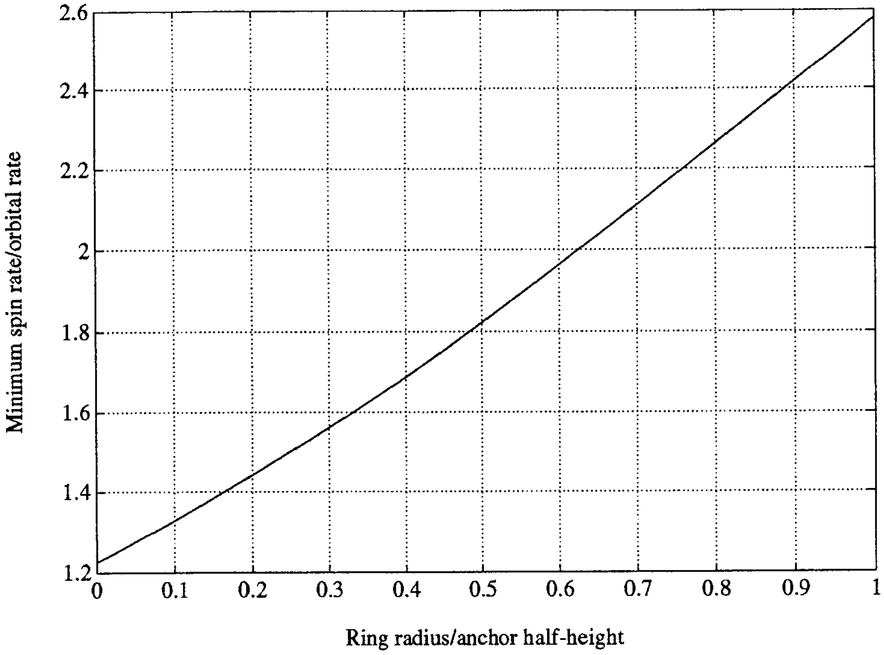

Figure 7 Minimum Spin Rate for Stability with Spine

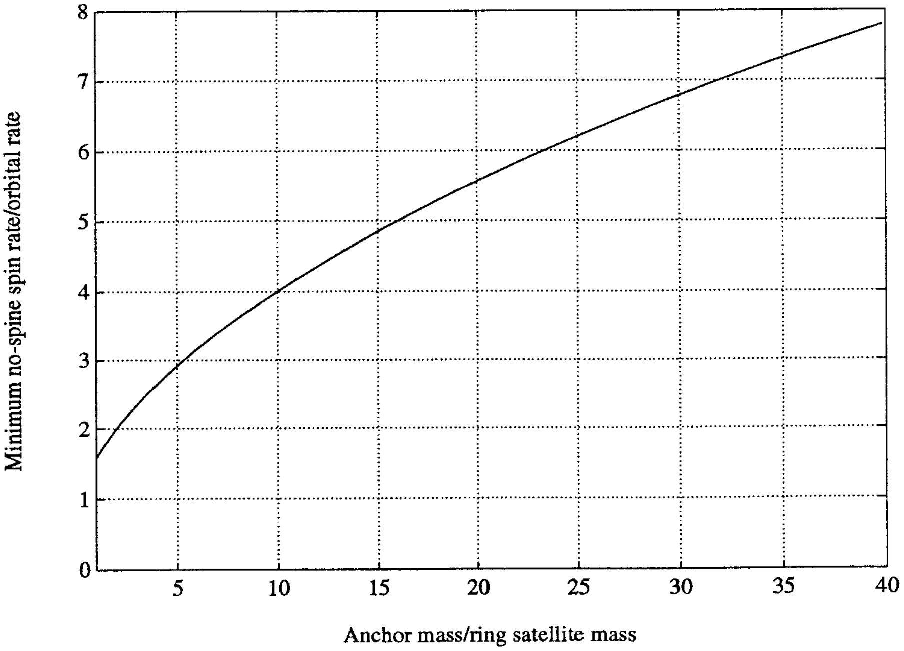

Figure 8 Minimum Spin Rate for No-Spine Stability

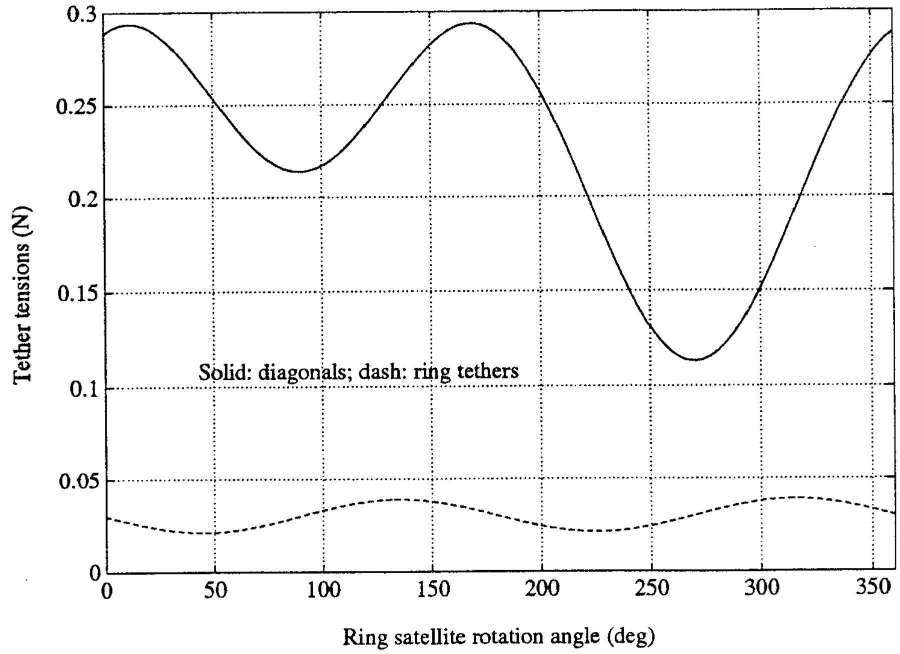

Figure 9 Tether Tensions for Exact Satellite Position-Keeping

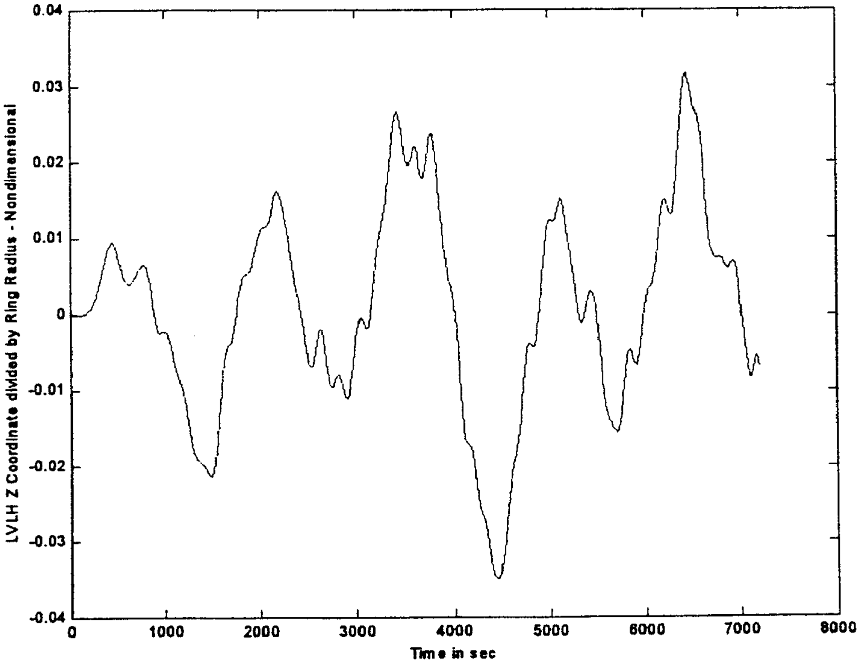

Figure 10 Ring Satellite z Position, No Tension Control