Robust line-of-sight stability and jitter compensation using spatio-temporal-filtering based control approaches

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ABSTRACT

A spatio-temporal filter (STF) based active vibration suppression technique is presented. The STF approach is intended for use for stability and jitter compensation for the UltraLITE Precision Deployable Experiment - a ground demonstration of a sparse array, deployable, large aperture, optical space telescope concept. This technique is well suited for control of complex, real-world structures because it requires little model information, autonomously accommodates sensor and actuator failures, is computationally efficient and the controller is easily updated to account for time varying system dynamics. An overview of the STF approach is given and experimental active vibration suppression results obtained on the Mirror Mass Simulator testbed at AFRL, Kirtland AFB are presented.

Keywords: vibration, suppression, deployable, optical, telescope, space, sensor, array, failure, spatio-temporal

1. INTRODUCTION

Structural vibration and structural dynamics issues are the cause of many difficulties in both commercial and military systems. It is a particularly significant source of problems in spacecraft and aircraft where weight restrictions may lead to structures that are inherently flexible. Vibration fundamentally limits the accuracy and resolution of weapons systems and space based sensing systems such as arrayed telescopes.

The Air Force Research Laboratory (AFRL), Kirtland Air Force Base, is conducting the UltraLITE Precision Deployable Experiment. This is a ground demonstration of a sparse array, deployable, large aperture, optical space telescope concept. While deployable optical arrays enable higher resolution to be achieved, the individual deployed mirror segments must be controlled to maintain position and jitter stability to less than 20 nanometers. Digital signal processing (DSP) capability available to implement controllers in a flight system is limited and reliability and robustness to failures and changing system dynamics is required.

While researchers and engineers have been investigating active vibration control approaches for many years there remain fundamental problems. The dynamic response of real-world structural systems is typically very complex. It is difficult to create robust, control oriented models of these systems, particularly if they are time varying which is often the case. To insure control stability and performance these models must include the effects of all dynamics in the frequency range of interest, meaning, typically, many structural vibration modes must be accurately modeled. In many real world applications this modeling effort can represent the majority of the total effort required to develop an active control strategy. Alternative approaches that do not require such an extensive and accurate system model would be preferable.

Spatio-Temporal Filtering (STF) is a practical and computationally efficient tool for controlling and monitoring the behavior of complex linear systems. STF based structural vibration control is a simple approach which is insensitive to the complexities, nonlinearities, time varying or unmodeled dynamics, component failures, and deviations from theoretical assumptions which often limit the usefulness of sophisticated control and estimation approaches. STF achieves the benefits of traditional active control approaches without many of the limitations. Some of the major advantages are;

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- The modeling requirement is greatly reduced. The only information required about the system is the pole values (frequency and damping) of the modes of interest. Modes that are not problematic can be ignored without risk of spill-over related instability. For monitoring and measuring applications non-target modes are rendered invisible and inconsequential.
- It is a robust method that inherently accommodates sensor and actuator failures. The system is autonomously reconfiguring, optimally utilizing remaining sensor and actuator resources to allow the mission to continue.
- Since the only system information utilized is a limited number of pole values, the controller is easily updated to accommodate time varying system dynamic characteristics.

The concept of Spatio-Temporal Filtering is simple; the dynamic response characteristics of a complex, multi-input, multi-output system with many structural modes can be decomposed into its fundamental components; simple first order systems described by a single pole value and input and output scaling coefficients. These simple, canonical components can easily be controlled or monitored independent of the complexities of the rest of the system.

STF is a sensor/actuator array based approach. It utilizes the sensing and actuation capability of multiple sensors and actuators in an integrated manner rather than individually. It is made feasible by recent developments in lower cost sensors and actuators and associated digital signal processing electronics. It is now feasible to use larger numbers of sensors and actuators for structural vibration control and monitoring.

This approach is unlike classical system identification/model based control. Rather than attempting to identify a complete and accurate model of a complex, possibly time varying system, STF based control attempts to make all of the system response except that associated with the modes of interest completely unobservable. This enables dealing with only the modes contributing to the phenomenon of interest – the rest of the system characteristics are of no concern and are not considered or unnecessarily accounted for; they are intentionally made invisible. This approach allows complex systems that are difficult to accommodate with conventional methods to be controlled and monitored.

Figure 1 illustrates using STF based control for vibration suppression of two critical low frequency modes of a complex system. The figure shows frequency response functions (FRFs) of a physical response, uncontrolled modal coordinates extracted with STF, controlled modal coordinate responses and the resulting closed loop physical system response.

The merit of this basic approach has been recognized by others in the past, however, a practical implementation has not previously been achieved. Meirovitch [1] proposed using strictly spatial or modal filters to implement Independent Modal Space Control (IMSC). The method required a distributed parameter model of the system in order to calculate modal filter coefficients. This is not possible for most systems of practical interest. The STF approach differs in two ways; 1) it filters in both space and time is to achieve higher accuracy with fewer sensors and; 2) a practical, reference model based approach is used to adaptively calculate spatio-temporal weighting coefficients. This approach requires minimal knowledge of the system, and given sufficient redundancy, corrects for sensor and actuator failures.

The feasibility of the STF concept was demonstrated under a Phase I SBIR contract awarded to Sheet Dynamics by the Ballistic Missile Defense Organization and managed by AFRL, Kirtland AFB. Sheet Dynamics has been awarded a Phase II SBIR contract to implement STF based vibration suppression on the UltraLITE Precision Deployable Ground Demonstration Experiment.

2. ADAPTIVE STF THEORY

In this section of the paper the origins of STF are discussed, STF theory and its application to structural vibration control is reviewed, and a reference model approach to adaptively calculating and updating STF coefficients is presented including the implications for sensor and actuator failure accommodation.

2.1 STF BACKGROUND

STF is an extension of modal, or spatial, filtering which has been investigated by the authors and other researchers for some time[1-7]. Modal filtering utilizes the characteristic that the dynamic response of any real structure is composed of a sum of individual modal responses, each behaving as a single-degree-of-freedom (SDOF) system and each having a particular response shape or eigenvector. The modal filter approach applies sets of scalar, spatial weighting coefficients to responses

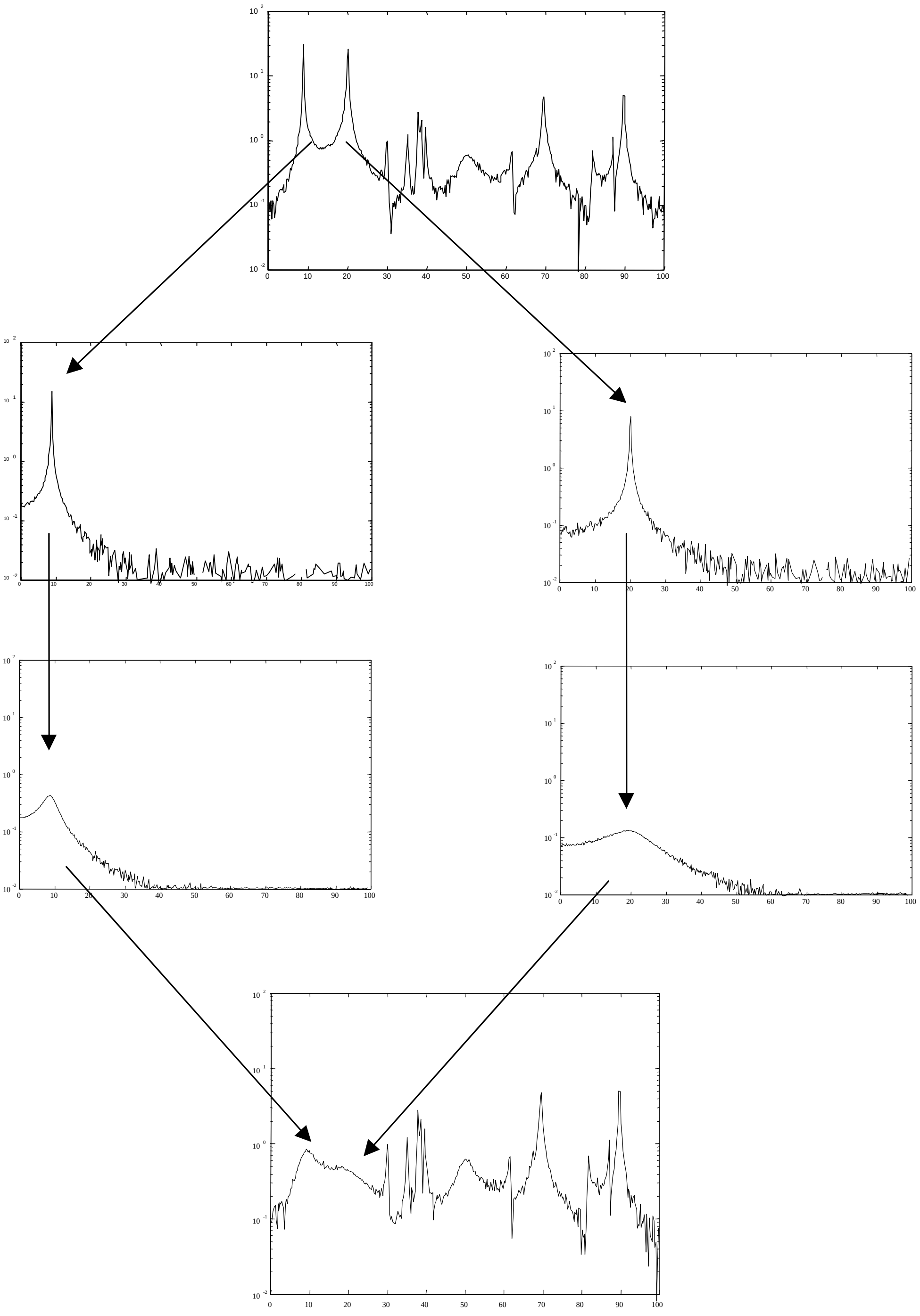

Figure 1: STF Based Structural Vibration Control: Uncontrolled Physical Response; STF Modal Coordinate Estimates – Uncontrolled; STF Modal Coordinate Estimates – Controlled; Resulting Controlled Physical Response

measured by each element of a sensor array to extract these individual canonical modal responses from the global response of the structure. Note the modal filter is **NOT** related to a conventional bandpass filter. Each channel of the modal filter has output across the entire frequency band, however, it is the output associated with just a single mode.

The basis for modal filtering is the standard modal coordinate transformation that is utilized to simplify the solution, understanding, and analysis of systems of linear differential equations. For simplicity consider the undamped, structural case where the common discrete system model consists of a second order linear differential equation with N by N matrix coefficients of mass and stiffness terms.

$$M\ddot{x} + Kx = f \qquad 1$$

Equation 1 may be solved for N linearly independent eigenvectors ϕ_r. Since the eigenvectors are linearly independent the system response, x, may be represented as a linear combination of the eigenvectors weighted by the canonical degrees of freedom, called modal coordinates, $\eta_r(t)$.

$$\begin{aligned} x(t) &= \sum_{r=1}^{N} \left[\phi_r \eta_r(t) \right] \\ &= \Phi \eta(t) \end{aligned} \qquad 2$$

For the physical case, where control and monitoring is being conducted on a structure, the response vector, $x(t)$, is measured with sensors located at corresponding physical locations and measurement directions. A modal filter is applied to the measured response data, $x(t)$, to extract the modal coordinate response(s), $\eta_i(t)$, of interest. To extract the modal coordinate response for the $i'th$ mode, a vector of spatial weighting coefficients, ψ_i, is sought which has the following characteristics;

$$\begin{aligned} \psi_i^T \phi_r &= 0 \quad i \neq r \\ &= 1 \quad i = r \end{aligned} \qquad 3$$

The inner product between the modal filter vector, ψ_i, and response vector, $x(t)$, is formed which is equivalent to forming a weighted average of the response signals measured at different locations on the structure.

$$\begin{aligned} \psi_i^T x(t) &= \psi_i^T \sum_{r=1}^{N} \left[\phi_r \eta_r(t) \right] \\ &= \psi_i^T \phi_i \eta_i(t) \\ &= \eta_i(t) \end{aligned} \qquad 4$$

The resulting scalar quantity is the modal coordinate response, $\eta_i(t)$, for the $i'th$ mode and the vector, ψ_i, is the associated modal filter vector. The above discussion holds for the damped case as well, both proportional and non-proportional [3].

2.1 SPATIO-TEMPORAL FILTERING

The spatio-temporal filter is a generalization of the spatial or modal filter which extends the capabilities by utilizing temporal information; two dimensional filtering in the space and time dimensions. This reduces the number of sensors required, allows dissimilar sensors to be integrated, and accommodates sensor dynamics.

In order to extract the modal coordinate response of interest with modal filters, the modal vectors, as sampled at the sensor locations, must be linearly independent [3]. This dictates that at least as many sensors as there are independent modes contributing to the measured response are required. Even with modern low cost sensors and DSP electronics this may be viewed as a disadvantage in some applications.

The modal filter estimates the modal coordinate response at time *k* by forming a weighted summation of sensor signals measured at different spatial locations at time *k*;

$$\hat{\eta}_k = \psi^T x_k \tag{5}$$

An N_i 'th order spatio-temporal filter also utilizes N_i past samples of the response information;

$$\hat{\eta}_k = \psi^T \begin{Bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-Nt} \end{Bmatrix} \tag{6}$$

This introduces a different N_i 'th order finite impulse response (FIR) or all-zero filter on each sensor channel. The FIR filters perform different functions depending on the specific implementation; pole-zero cancellation if spatial resolution alone is insufficient to separate modes, accommodating the relative phase between sensors caused by complex (non-proportional damping) modes, selective differentiation if a nonhomogeneous sensor array is used, and correction for some sensor dynamics. Most likely a combination of the above characteristics will be manifested in the temporal filter component of the STF.

2.3 REFERENCE MODEL SOLUTION

This section introduces a reference model approach for adaptively calculating and updating the STF filter coefficients with very little a priori information. This enables the method to be applied to any arbitrarily complex real-world structure and accommodate sensor and actuator failures in a manner which it transparent to control and monitoring algorithms.

The structure of the STF filter estimation problem is similar to other estimation problems in that an error is defined which is a function of the parameters (in this case STF filter vector coefficients) to be estimated. The parameters are estimated by minimizing the error. As with other estimation problems, different solution methods may be employed to minimize the error to arrive at a solution that is optimal in some sense.

The subscript denoting mode number on the variables is dropped. The development is applicable to any single mode. Discrete time is assumed with the subscript now indicating sample number.

For clarity, first consider a single input modal filter (no temporal information) estimation problem. An error is defined which is the difference between the true modal coordinate, η_k, and the modal coordinate estimated by the spatial filter, $\hat{\eta}_k$, at time k.

$$\begin{aligned} e_k &= \eta_k - \hat{\eta}_k \\ &= \eta_k - \psi_k^T x_k \end{aligned} \tag{7}$$

The true modal coordinate, however, is not known. Indeed, estimating η_k is the purpose of the modal filter or STF. A reference modal coordinate, $\eta_k^{(r)}$, which is highly correlated with the true modal coordinate may be generated by driving a SDOF reference system constructed from only the pole of the mode of interest. The first order, discrete time reference system is;

$$\eta_{k+1}^{(r)} = z_\lambda \eta_k^{(r)} + f_k \tag{8}$$

z_λ is the Z domain pole $z_\lambda = e^{\lambda \Delta t}$. In this case the driving force, f_k, is the measured control force. The reference modal coordinate is then used in Equation 7 in place of the true modal coordinate to calculate the error. The solution problem is to minimize this error over a number of time steps to estimate ψ. The structure of this problem is illustrated in Figure 2.

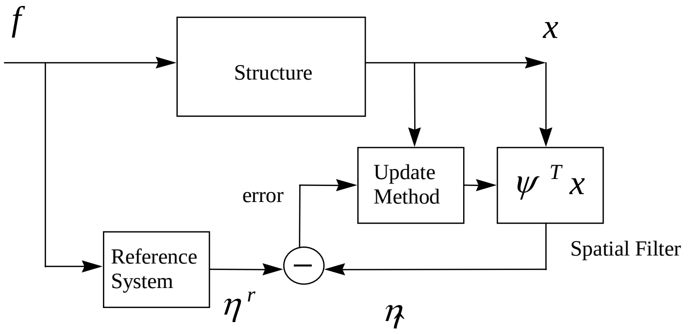

Figure 2: Structure of Modal Filter Estimation Problem

For the multi-input case, the total modal coordinate response of each mode is due to multiple input forces. The effect of each input is described by an unknown vector of, possibly, complex force appropriation coefficients, l (also called modal participation vectors) [3]. The reference model becomes;

$$\eta_{k+1}^{(r)} = z_\lambda \eta_k^{(r)} + l^T f_k \qquad 9$$

where f_k is now a vector of applied forces. In this case the reference system is driven with a modal force consisting of the sum of the input forces weighted by the force appropriation vector coefficients. An equivalent reference modal coordinate may be generated by driving N_i reference models;

$$\begin{aligned} \eta_{k+1}^{(r1)} &= z_\lambda \eta_k^{(r1)} + f_k^{(1)} \\ &\vdots \\ \eta_{k+1}^{(rN_i)} &= z_\lambda \eta_k^{(rN_i)} + f_k^{(N_i)} \end{aligned} \qquad 10$$

by the unweighted N_i forces and using the force appropriation vector to form a weighted average of the Ni reference modal coordinates.

$$\begin{aligned} \eta_k^{(r)} &= l^T \begin{Bmatrix} \eta_k^{(r1)} \\ \vdots \\ \eta_k^{(rN_i)} \end{Bmatrix} \\ &= l^T \eta_k^r \end{aligned} \qquad 11$$

Note the distinction between $\eta_k^{(r)}$ which is the scalar modal coordinate and η_k^r which is a vector of partial modal coordinate responses associated with the individual input forces. The general spatio-temporal filter error with both input and output temporal filtering is then;

$$
\begin{aligned}
e_k &= \eta_k^{(r)} - \hat{\eta}_k \\
&= l^T \begin{bmatrix} \eta_k^r \\ \eta_{k-1}^r \\ \vdots \\ \eta_{k-Nti}^r \end{bmatrix} - \psi^T \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \\ x_{k-Nto} \end{bmatrix} \\
&= \begin{Bmatrix} \psi \\ l \end{Bmatrix}^T \begin{bmatrix} -x_k \\ \vdots \\ -x_{k-Nto} \\ \eta_k^r \\ \vdots \\ \eta_{k-Nti}^r \end{bmatrix}
\end{aligned}
\tag{12}
$$

A trivial solution which must be avoided is the zero error solution where both the response STF filter vector, ψ, and force appropriation vector, l, are zero. This may be accomplished by artificially normalizing one of the coefficients to, for instance, unity. This has the draw back that, if the coefficient is physically close to zero amplitude, the problem is very ill conditioned and an inaccurate solution results. The preferable solution is to impose a norm constraint on the solution vector where, for instance, the norm of the solution vector is constrained to unity. Additional norm constraint considerations arise when the updating the STF inside a control loop. In this case the norm constraint must be applied only to the response weighting filter coefficients.

Different solution methods have been utilized for the STF filter estimation method. Of most interest are adaptive, on-line methods since they recover from sensor and actuator failure. In the event of a sensor failure the response related STF filter coefficients update to continue to optimally estimate the modal coordinate response with the remaining sensing capacity. Provided sufficient sensing capability remains to estimate the modal coordinates, a controller utilizing these outputs for feedback control would be unaffected by the sensor failure. Multiple input modal controllers apply a control force vector which is either l, the force appropriation vector component of the STF or a direct function of it. The “force” driving the reference models is generally the command to the control actuators. An actuator failure is reflected in the associated force appropriation coefficient estimated by the STF and is inherently accommodated.

To date two different real-time adaptive STF update methods have been evaluated in real-time implementation using the USAF Phillips Lab dSpace data acquisition and signal processing hardware; Least Mean Squares (LMS) [8] and Recursive Least Squares (RLS) [9].

Major benefits of the RLS approach are superior convergence speed and accuracy. It has a higher computational demand, however, multiple modes can be estimated with little additional computational burden. The additional computational burden to update a STF with Ns sensors and Nt time taps is approximately 2*Ns*Nt floating point multiplies and adds.

3. STF BASED CONTROL

The STF is not, in itself, a vibration controller. However, design of very effective STF based, multiple-input, multiple-output, active vibration suppression controllers generally entails selection of merely a single scalar control gain parameter for each mode to be controlled.

The reference model utilized to adaptively update the STF coefficients can take the form of a position, velocity or acceleration output model. The adaptive STF will attempt to form a modal coordinate output that matches the reference model. For active vibration suppression a modal coordinate velocity output is often desired to utilize directly for rate feedback control.

In this case the control command for each mode consists of;

$$f_c^{(i)} = \hat{\dot{\eta}}^{(i)}\alpha^{(i)}v^{(i)} \qquad 13$$

where $f_c^{(i)}$ is the control command (generally a force command) vector output to control the i'th mode, $\hat{\dot{\eta}}^{(i)}$ is the estimate of the modal coordinate velocity of the i'th mode generated by the STF, $\alpha^{(i)}$ is the control gain and $v^{(i)}$ is the forcing vector. The theoretical control gain required to achieve a certain level of damping can be calculated, however, in practice it is often more effective to manually adjust control gain.

A number of considerations may effect the choice of forcing vector. In general the forcing vector should be chosen to project strongly on the force appropriation vector (FAV). This is desired since the resulting modal control force is the inner product of the FAV and the control force vector, $l^{(i)^T} f_c^{(i)}$. A good choice of control force vector to maximize this inner product is the FAV vector itself. Since the STF automatically generates the FAV vector it is the logical choice. For each modal controller the control force is;

$$f_c^{(i)} = \hat{\dot{\eta}}^{(i)}\alpha^{(i)}l^{(i)} \qquad 14$$

Control design, then, consists of choosing the control gain, $\alpha^{(i)}$, for each controlled mode. Multiple modal controllers are run in parallel to control multiple modes. In this case the physical control force command is the sum of the individual modal control forces.

Active vibration suppression experiments were conducted on the Mirror Mass Simulator (MMS) test bed at AFRL, Kirtland AFB. The MMS consists of a dummy optical mirror supported on three piezoelectric stack actuators with three nearly co-located optical interferometers for position measurement. The entire assembly is mounted on an optical bench. The dummy mirror is an aluminum honeycomb structure with size, mass and stiffness characteristics similar to those of the lightweight composite mirrors planned for the UltraLITE deployable optical telescope.

Controllers were implemented using dSpace™ hardware hosted by a PC computer. Using the Matlab™, Real Time Workshop™, controllers designed and simulated in Simulink™ were complied, downloaded and run in the dSpace hardware. The general structure of the controller and system is illustrated in Figure 3.

Initially the system was driven (through the piezo-stack actuators) with a random excitation generated within the dSpace system in order to adaptively identify STF filter coefficients. The same internal digital excitation signal which is output through the dSpace DACs to drive the physical system is also used internally to drive the internal SDOF reference models. The STF is adaptively calculated such that it's output matches the output of the reference system.

This approach has advantages that are not immediately obvious. The system input-output characteristics which the control system sees include not only the structural characteristics of the plant but also the characteristics of the dSpace DACs (and output filters?), piezo-stack amplifiers, piezo-stack actuators, interferometer sensors and signal conditioners, dSpace ADCs (and input/anti-aliasing filters?) and possibly other miscellaneous effects. Since the adaptive STF is trying to make the dynamics of the path between the internal digital output command (which also drives the reference model) and the internal output of the STF match the dynamics of the SDOF reference system, which does not include these effects, it inherently attempts to compensate for all of these effects. The extent to which it can be successful at this task is not fully understood. It depends on the number of time taps used, the number of sensors and modes in the frequency band, and the extent to which the loop dynamics can be inverted with an FIR filter (the temporal characteristics of the STF are an FIR filter).

A five-mode controller was implemented on the MMS utilizing the three piezoelectric stack actuators and the three interferometers as control and sensing devices. Illustrating the simplicity and practical nature of the STF approach, this was done in a period of one and a half days which including becoming familiar with the MMS test bed and conducting all required system identification, controller design, implementation and testing.

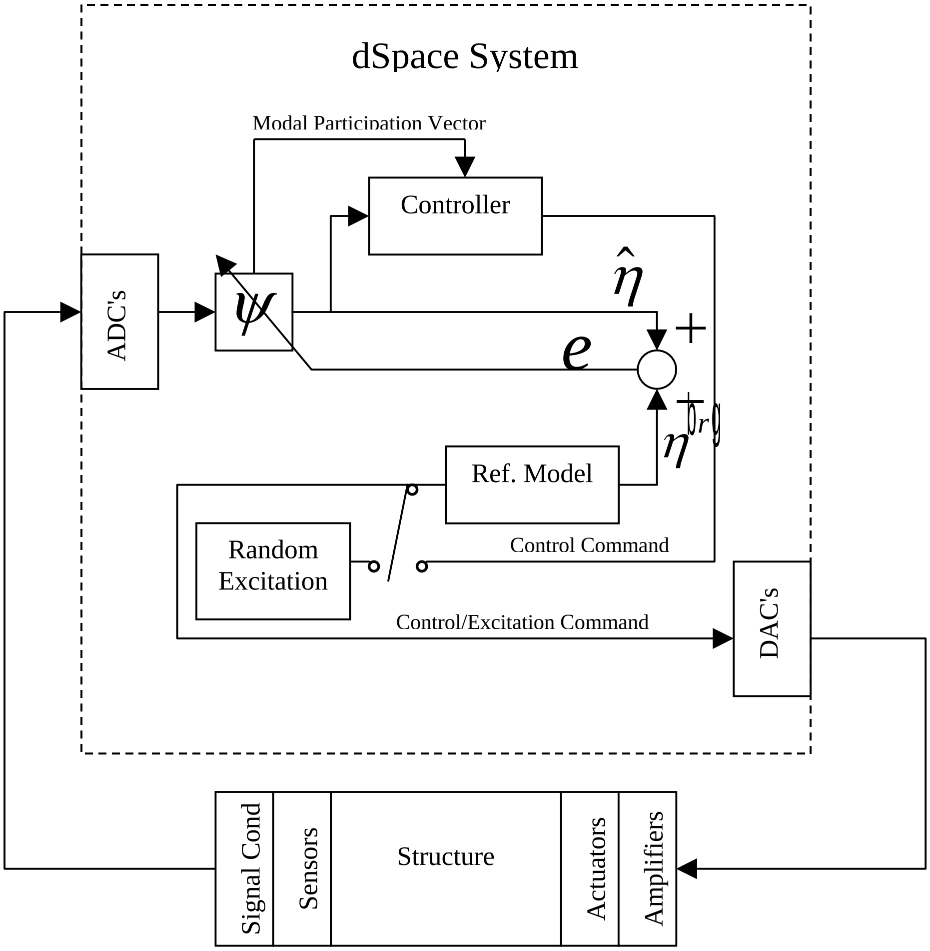

Figure 3: Structure of STF Based Controller

To evaluate the effectiveness of the controller an electromagnetic shaker was used to apply a fourth force input to the MMS as a disturbance input. Frequency response functions (FRFs) were calculated between the disturbance force and the three interferometer responses for the open loop case (O/L – controller turned off) and the closed loop case (C/L – controller turned on).

Figures 4 through 6 show overlaid O/L and C/L FRFs for the three interferometer responses. The five controlled modes are the modes at roughly 74, 84, 153, 163 and 171 Hertz. As the plots demonstrate, resonant peaks are reduced in amplitude by approximately 20 db or a factor of 10. Note there is some spillover into an uncontrolled mode at approximately 190 Hertz. Modes above this frequency are unaffected by the controller.

These successful experimental active vibration suppression results were obtained with an adaptive LMS STF update algorithm that was not well suited to the multiple input case. The resulting STF filters did not accurately estimate the modal coordinate velocities of interest – there was significant corruption from other modes resulting in the control spillover into the 190 Hertz mode. A recursive least squares update algorithm with norm constraint and covariance matrix resetting has since been implemented with significantly superior performance.

Though space limitations do not permit presentation of results, experiments demonstrating the sensor failure accommodation capability of STF were also conducted at AFRL, Kirtland AFB.

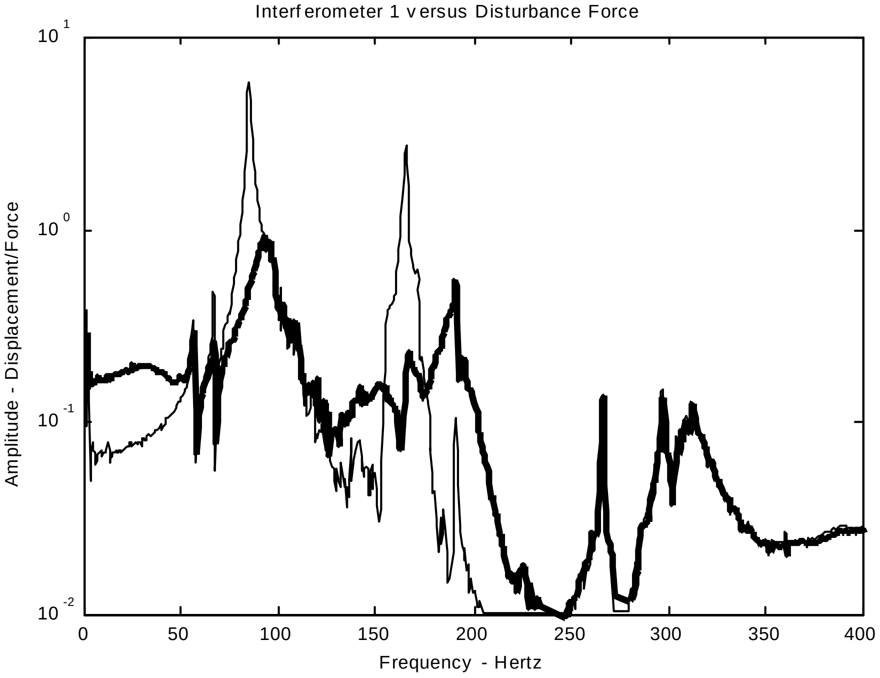

Figure 4: Mirror Mass Simulator Active Vibration Suppression Results – Thin line is FRF between disturbance force and Interferometer 1 with controller turned off; Thick line is with controller turned on.

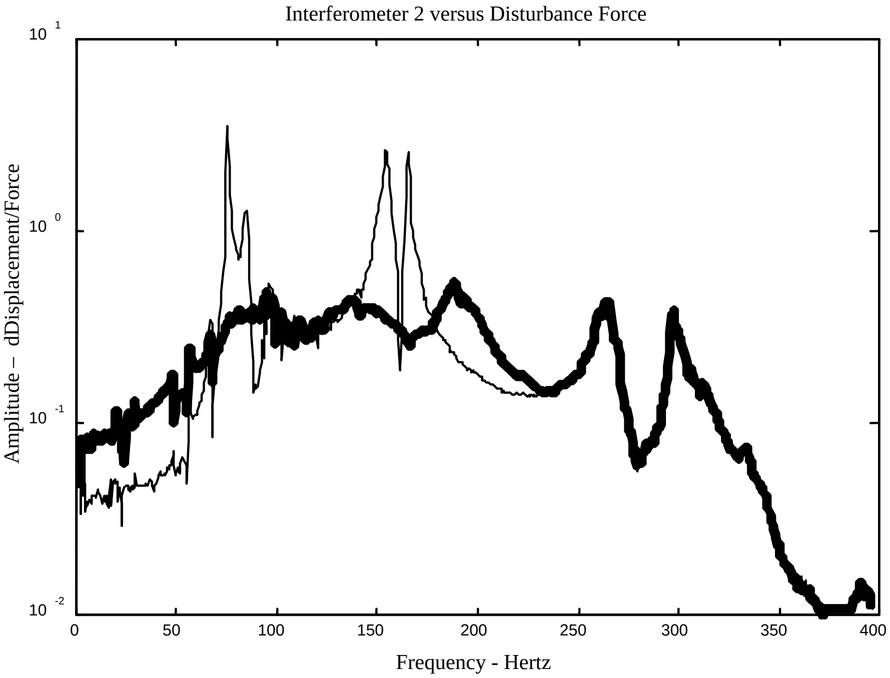

Figure 5: Mirror Mass Simulator Active Vibration Suppression Results – Thin line is FRF between disturbance force and Interferometer 2 with controller turned off; Thick line is with controller turned on.

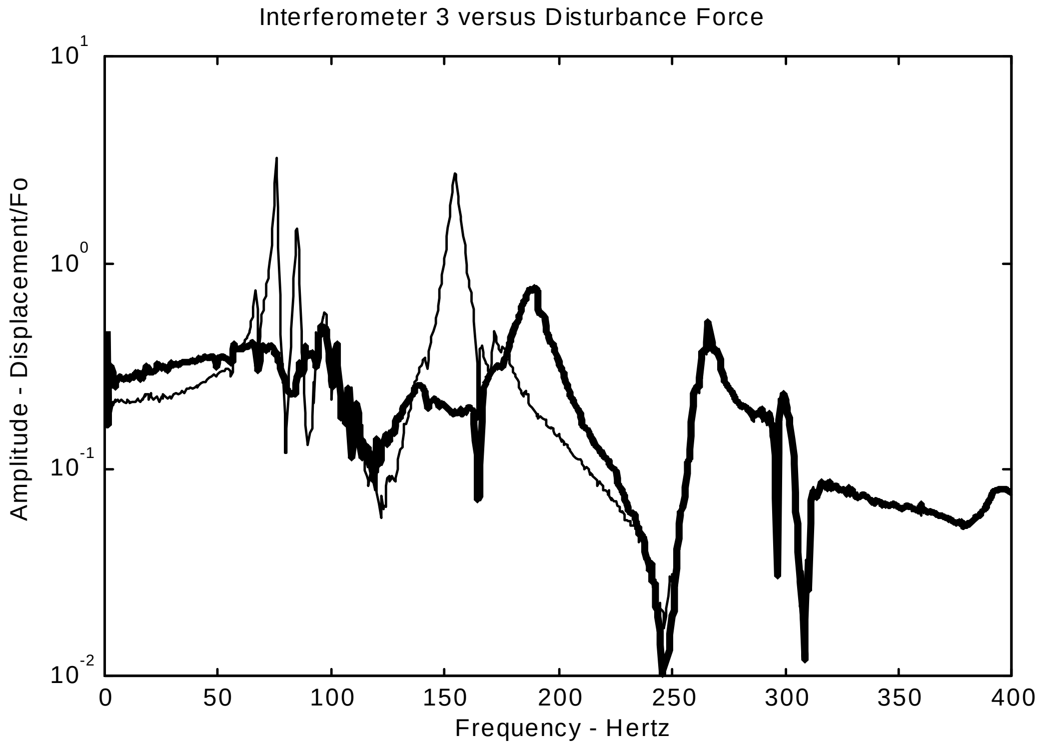

Figure 6: Mirror Mass Simulator Active Vibration Suppression Results – Thin line is FRF between disturbance force and Interferometer 3 with controller turned off; Thick line is with controller turned on.

This control approach will continue to be refined through the course of Sheet Dynamic's Phase II SBIR contract to meet the stability and jitter suppression requirements of the UltraLITE sparse array optical space telescope project.

4. CONCLUSIONS

Experimental implementation on the AFRL Kirtland AFB Mirror Mass Simulator has demonstrated the effectiveness and practical nature of the STF control approach for active vibration suppression on space oriented structures. The approach will continue to be refined for implementation on the UltraLITE Precision Deployable Experiment - a ground demonstration of a sparse array, deployable, large aperture, optical space telescope concept.

5. ACKNOWLEDGEMENTS

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